Doctopic: Functional Analysis ARTICLE IN PRESS YJMAA:22410

J. Math. Anal. Appl. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Near-epoch dependence in Riesz spaces

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A R T I C L E I N F O

Article history:
Received 16 March 2018
Available online xxxx
Submitted by D. Khavinson

Keywords:
Vector lattices
Riesz space
Conditional expectation operators
Mixing processes
Mixingales
Near-epoch dependence

A B S T R A C T

The abstraction of the study of stochastic processes to Banach lattices and vector lattices has received much attention by Grobler, Kuo, Labuschagne, Stoica, Troitsky and Watson over the past fifteen years. By contrast mixing processes have received very little attention. In particular mixingales were generalized to the Riesz space setting in Kuo et al. (2013) [12]. The concepts of strong and uniform mixing as well as related mixing inequalities were extended to this setting in Kuo et al. (2017) [11]. In the present work we formulate the concept of near-epoch dependence for Riesz space processes and show that if a process is near-epoch dependent and either strong or uniform mixing then the process is a mixingale, giving access to a law of large numbers. The above is applied to autoregressive processes of order 1 in Riesz spaces.

1. Introduction

The 1962 paper of Ibragimov [8, pages 370–371] considers the variance and limiting cumulative distribution of functions of stationary strong mixing processes. Here the expectation of the deviation of the process from the time symmetric conditional expectations of the process with respect to the events in the time window is assumed to be summable over all window sizes. Billingsley [2, Section 21] in his 1967 monograph developed on the ideas of Ibragimov. He worked in L^2, as opposed to L^1 used by Ibragimov, and was able to give a functional central limit theorem for such processes. Further relaxing the assumptions on these processes, McLeish [15, page 837] was still able to obtain a strong law of large numbers for these processes as an application of his theory of mixingales. In addition, it was McLeish who first used the term ‘epoque’ in this context. Gallant and White [5] generalized the theory to arrays of processes in L^p, they also introduced

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[1] Supported in part by NRF grant number CSUR160503163733.
[2] Supported in part by the Centre for Applicable Analysis and Number Theory and by NRF grant number IFR170214222646 with grant no. 109289.

https://doi.org/10.1016/j.jmaa.2018.07.019
0022-247X/

Please cite this article in press as: W.-C. Kuo et al., Near-epoch dependence in Riesz spaces, J. Math. Anal. Appl. (2018), https://doi.org/10.1016/j.jmaa.2018.07.019

the term 'near-epoch dependence' and presented a unified theory thereof. We refer the reader to Davidson [3, page 261] for a further history of this development.

For the reader's convenience we recall here the definition of near-epoch dependence given by Davidson in [3]. A sequence of integrable random variables $(X_i)_{i\in\mathbb{Z}}$ in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is said to be near-epoch dependent in L^p-norm on the sequence $(V_i)_{i\in\mathbb{Z}}$ in $(\Omega, \mathcal{F}, \mathbb{P})$ if, for $\mathcal{F}_{i-m}^{i+m} = \sigma(V_{i-m}, \dots, V_{i+m})$, the σ-algebra generated by $V_{i-m}, \dots, V_{i+m}$,

$$\|X_i - \mathbb{E}[X_i|\mathcal{F}_{i-m}^{i+m}]\|_p \leq d_i v_m$$

where $v_m \to 0$ as $m \to \infty$ and $(d_i)_{i\in\mathbb{Z}}$ is a sequence of positive constants.

Schaefer [16], Stoica [17,18] and Troitsky [20] considered the extension of the concepts of conditional expectation and martingales to Banach lattices. See [7,19,21] for some recent developments in this area. In 2004, Kuo, Labuschagne and Watson [9] generalized these concepts to vector lattices (Riesz spaces). de Pagter and Grobler in [4] introduced the extension of a conditional expectation operator on a probability space to its natural domain. This extension was generalized to the Riesz space setting by Kuo, Labuschagne and Watson in [10] and enabled the definition of the generalized L^p spaces in [1,11,14]. In particular, the natural domain of a Riesz space conditional expectation operator, T, is identified with the generalized L^p space $\mathcal{L}^1(T)$ and $T|\cdot|$ defines an $R(T)_+$ valued norm on $\mathcal{L}^1(T)$, see [11], where $\mathcal{L}^\infty(T)$ and its $R(T)_+$ valued norm are also defined. Building on this structure, using functional calculus, similar constructions can be made for $p \in (1, \infty)$, see [1]. Critical in all of the above is that $R(T)$ is a universally complete f-algebra, see [11]. The final piece needed in this context is Jensen's inequality in Riesz spaces, developed by Grobler in [6].

In [24] a Douglas–Andô–Radon–Nikodým theorem on Riesz spaces with a conditional expectation operator was proved. This gives the necessary tools for building and identifying conditional expectation operators on Riesz spaces, required in the study of Markov, [22,23] and mixing processes, [11]. The study of mixing processes in Riesz spaces began with the formulation of mixingales in Riesz spaces in [12], where a law of large numbers was proved for Riesz space mixingales. In [11] strong and uniform mixing processes were formulated in Riesz spaces and mixing inequalities proved for them.

In this work we introduce the concept of near-epoch dependence for Riesz space processes and show that if a process is near-epoch dependent and either strong or uniform mixing then the process is a mixingale, giving access again to a law of large numbers. Finally the above is applied to autoregressive processes of order 1 in Riesz spaces.

In Section 2 we provide the required background material on the generalized L^p spaces. In Section 3 we recall for the readers' convenience the necessary results on mixingales and mixing processes in Riesz spaces. In Section 4 we formulate near-epoch dependence for Riesz space processes and prove that a near-epoch dependent process which is strong or uniform mixing is a mixingale, hence showing that such processes obey a law of large numbers. In Section 5 we apply the results of Section 4 to autoregressive processes of order 1. These results lead to new results when applied to the classical probability setting, for this we refer the reader to the last section of [11].

2. Preliminaries

In this section we outline the prerequisite material relating to stochastic processes in Riesz spaces necessary for this paper. For a more thorough examination of the contents of this section, see, for example, [9,10]. In addition, we will define the $\mathcal{L}^2(T)$ space and present the corresponding vector-valued norm, which will be defined with respect to the conditional expectation operator on the space.

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