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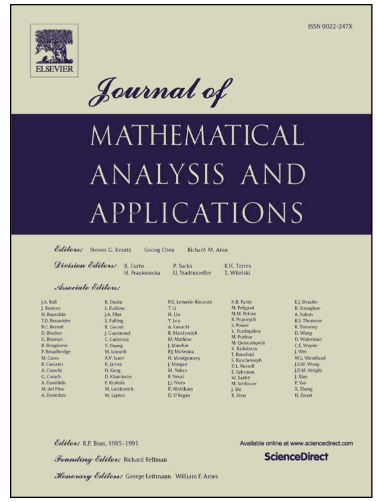

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ASYMPTOTIC MEAN VALUE PROPERTIES FOR FRACTIONAL ANISOTROPIC OPERATORS

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Abstract. We obtain an asymptotic representation formula for harmonic functions with respect to a linear anisotropic nonlocal operator. Furthermore we get a Bourgain-Brezis-Mironescu type limit formula for a related class of anisotropic nonlocal norms.

Contents

1. Introduction

This paper presents an asymptotic mean value property for harmonic functions for a class of anisotropic nonlocal operators. To introduce the argument, we notice that as known from elementary PDEs facts, a C^2 function $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ is harmonic in Ω (i.e. it holds that $\Delta u = 0$ in Ω) if and only if it satisfies the mean value property, that is

$$u(x) = \fint_{B_r(x)} u(y)dy, \qquad \text{whenever } B_r(x) \subset \Omega.$$

As a matter of fact, this condition can be relaxed to a pointwise formulation by saying that $u \in C^2(\Omega)$ satisfies $\Delta u(x) = 0$ at a point $x \in \Omega$ if and only if

$$u(x) = \fint_{B_r(x)} u(y)dy + \mathfrak{o}(r^2), \qquad \text{as } r \to 0. \tag{1.1}$$

This asymptotic formula holds true also in the viscosity sense for any continuous function. A similar property can be proved for quasi-linear elliptic operators such as the p-Laplace operator $-\Delta_p u$ in the asymptotic form, as the radius r of the ball vanishes. More precisely, Manfredi, Parviainen and Rossi proved in [20] that, if $p \in (1, \infty]$, a continuous function $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ is p-harmonic in Ω in viscosity sense if and only if

$$\varphi(x) \geq (\leq) \, \frac{p-2}{2p+2n}\Big(\max_{B_r(x)} \varphi + \min_{B_r(x)} \varphi\Big) + \frac{2+n}{p+n}\fint_{B_r(x)} \varphi(y)dy + \mathfrak{o}(r^2), \tag{1.2}$$

for any $\varphi \in C^2$ such that $u - \varphi$ has a strict minimum (strict maximum for $\leq$) at $x \in \overline{\Omega}$ at the zero level. Notice that formula (1.2) reduces to (1.1) for $p = 2$. Formula (1.2) holds in the classical sense for smooth functions, at those points $x \in \overline{\Omega}$ such that $\nabla u(x) \neq 0$. On the other hand, the

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