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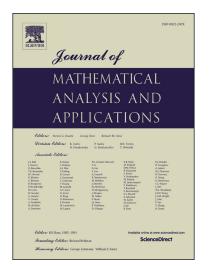
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ACCEPTED MANUSCRIPT

INTERPOLATION BETWEEN HÖLDER AND LEBESGUE SPACES WITH APPLICATIONS

ANASTASIA MOLCHANOVA, TOMÁŠ ROSKOVEC, AND FILIP SOUDSKÝ

In memory of Václav Nýdl.

ABSTRACT. Classical interpolation inequality of the type $||u||_X \leq C||u||_Y^{\theta}||u||_Z^{1-\theta}$ is well known in the case when X, Y, Z are Lebesgue spaces. In this paper we show that this result may be extended by replacing norms $||\cdot||_Y$ or $||\cdot||_X$ by suitable Hölder semi-norm. We shall even prove sharper version involving weak Lorentz norm. We apply this result to prove the Gagliardo–Nirenberg inequality for a wider scale of parameters.

1. INTRODUCTION AND MAIN RESULT

The classical Sobolev embedding theorem claims that if $1 \le p < n$ then for any weakly differentiable function $u \in WL^p$ one has

$$\|u\|_{p^*} \le C \|\nabla u\|_p,$$

where $p^* = \frac{np}{n-p}$ and C > 0 is independent of u. If p > n then by the Morrey lemma, for the continuous representative the following holds

$$\|u\|_{\mathcal{C}^{0,1-\frac{n}{p}}} \le C \|\nabla u\|_p.$$

These are classical results which can be found, for instance, in classical books [2] or [9]. Following the notation of L. Nirenberg [20, Lecture II], consider the extended norm for $-\infty < \frac{1}{p} < \infty$.

Definition 1.1. For $p \in (0, \infty]$ define

$$||u||_p = \left(\int_{\mathbb{R}^n} |u|^p dx\right)^{\frac{1}{p}};$$

and

 $||u||_{\infty} = \operatorname{esssup}_{x \in \mathbb{R}^n} |u(x)|.$

For p < 0 set numbers s and \tilde{p} by s = [-n/p] (where $[\alpha]$ stands for the integer part of α), $n/\tilde{p} = s + n/p$, and define

(1)
$$\begin{aligned} \|u\|_p &= \|\nabla^s u\|_{\tilde{p}}, \qquad \text{if } -\infty < \tilde{p} < -n, \\ \|u\|_p &= \|\nabla^s u\|_{\infty}, \qquad \text{if } s = -n/p, \end{aligned}$$

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