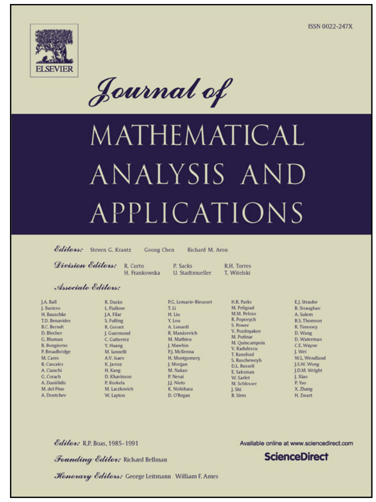
Accepted Manuscript



Interpolation between Hölder and Lebesgue spaces with applications

Anastasia Molchanova, Tomáš Roskovec, Filip Soudský

PII: S0022-247X(18)30468-2
DOI: https://doi.org/10.1016/j.jmaa.2018.05.067
Reference: YJMAA 22299

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 23 January 2018

Please cite this article in press as: A. Molchanova et al., Interpolation between Hölder and Lebesgue spaces with applications, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2018.05.067

INTERPOLATION BETWEEN HÖLDER AND LEBESGUE SPACES WITH APPLICATIONS

ANASTASIA MOLCHANOVA, TOMÁŠ ROSKOVEC, AND FILIP SOUDSKÝ

In memory of Václav Nýdl.

Abstract. Classical interpolation inequality of the type $\|u\|_X \le C\|u\|_Y^{\theta}\|u\|_Z^{1-\theta}$ is well known in the case when X, Y, Z are Lebesgue spaces. In this paper we show that this result may be extended by replacing norms $\|\cdot\|_Y$ or $\|\cdot\|_X$ by suitable Hölder semi-norm. We shall even prove sharper version involving weak Lorentz norm. We apply this result to prove the Gagliardo–Nirenberg inequality for a wider scale of parameters.

1. Introduction and main result

The classical Sobolev embedding theorem claims that if $1 \le p < n$ then for any weakly differentiable function $u \in WL^p$ one has

$$\|u\|_{p^*} \le C\|\nabla u\|_p,$$

where $p^* = \frac{np}{n-p}$ and $C > 0$ is independent of u. If $p > n$ then by the Morrey lemma, for the continuous representative the following holds

$$\|u\|_{\mathcal{C}^{0,1-\frac{n}{p}}} \le C\|\nabla u\|_p.$$

These are classical results which can be found, for instance, in classical books [2] or [9]. Following the notation of L. Nirenberg [20, Lecture II], consider the extended norm for $-\infty < \frac{1}{p} < \infty$.

Definition 1.1. For $p \in (0, \infty]$ define

$$\|u\|_p = \left(\int_{\mathbb{R}^n} |u|^p dx\right)^{\frac{1}{p}};$$

and

$$\|u\|_\infty = \operatorname{esssup}_{x\in\mathbb{R}^n} |u(x)|.$$

For $p < 0$ set numbers s and $\tilde{p}$ by $s = [-n/p]$ (where $[\alpha]$ stands for the integer part of α), $n/\tilde{p} = s + n/p$, and define

$$\begin{aligned} \|u\|_p &= \|\nabla^s u\|_{\tilde{p}}, && \text{if } -\infty < \tilde{p} < -n, \\ \|u\|_p &= \|\nabla^s u\|_\infty, && \text{if } s = -n/p, \end{aligned} \tag{1}$$

2010 *Mathematics Subject Classification.* Primary: 46E30, 46E35; Secondary: 35J47.
Key words and phrases. Interpolation, Lorentz spaces, Hölder spaces, inequality, Sobolev spaces.
The second author was supported by EF–IGS2017–Rost-IGS04B1.
The third author was supported by EF–IGS2017–Soudský–IGS07P1.