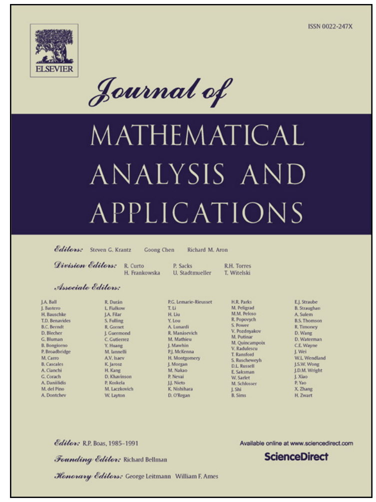
Accepted Manuscript



Variations on the Boman covering lemma

J.M. Aldaz

PII: S0022-247X(18)30609-7
DOI: https://doi.org/10.1016/j.jmaa.2018.07.027
Reference: YJMAA 22418

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 29 September 2017

Please cite this article in press as: J.M. Aldaz, Variations on the Boman covering lemma, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2018.07.027

VARIATIONS ON THE BOMAN COVERING LEMMA

J. M. ALDAZ

Abstract. We explore some variants of the Boman covering lemma, and their relationship to the boundedness properties of the maximal operator. Let $1 < p < \infty$ and let q be its conjugate exponent. We prove that the strong type (q, q) of the uncentered maximal operator, by itself, implies certain generalizations of the Boman covering lemma for the exponent p, and in turn, these generalizations entail the weak type (q, q) of the centered maximal operator. We show by example that it is possible for the uncentered maximal operator to be unbounded for all $1 < s < \infty$, while the conclusion of the lemma holds for every $1 < p < \infty$; thus, the latter condition is much weaker. Also, the boundedness of the centered maximal operator entails weak versions of the lemma.

1. Introduction

Often called "a useful lemma" or a "rather well-known lemma", the Boman covering lemma appears for the first time in [Bom], but [Boj] seems to be the standard reference (I have not have access to [Bom], a technical report). Hence, I will refer to [Boj] in what follows.

Closely connected to what nowadays is called the Boman chain condition, the lemma has been used to extend inequalities of Sobolev and Poincaré type to a wide class of domains (both in euclidean spaces and in several kinds of metric measure spaces) to explore the fine properties of functions belonging to different function spaces, and even to seemingly unrelated areas, such as complex dynamics. Among the many papers that could be cited here, we mention [IwNo], [Ch], [Ch1], [ChWh], [IhVa], [HeKo], [KoShSt].

In view of its usefulness it seems advisable, first, to name the lemma after its author, and second, to explore possible variations and generalizations, studying their relationships with the boundedness properties of maximal operators, both centered and uncentered.

Given $1 < p < \infty$ and Lebesgue measure on $\mathbb{R}^d$, the Boman covering Lemma tells us that the overlap of cubes (with sides parallel to the coordinate axes) when we increase their radii, grows in a controlled manner in the L^p norm (see Lemma 3.1 below for the precise statement). The result is an immediate consequence of Hölder's inequality and the boundedness of the uncentered maximal operator. Of course, the same proof works for metric measure spaces with a doubling measure.

Here we study alternative formulations of the lemma where doubling is not assumed. After some fairly standard definitions in Section 2, Section 3 considers variants of Boman's lemma.

2000 *Mathematical Subject Classification.* 42B25.

The author was partially supported by Grant MTM2015-65792-P of the MINECO of Spain, and also by by ICMAT Severo Ochoa project SEV-2015-0554 (MINECO).