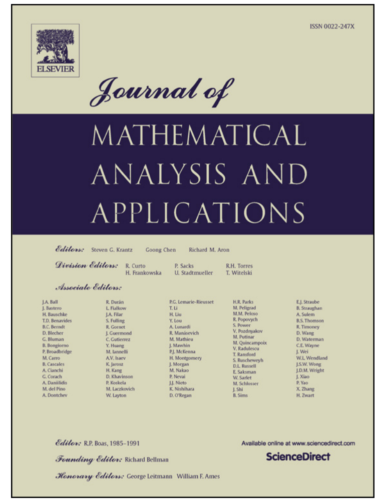

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Asymptotics of Sobolev orthogonal polynomials for Hermite (1,1)-coherent pairs

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Abstract

In this paper we will discuss asymptotic properties of monic polynomials $\{S_n^{\lambda}(x)\}_{n\geq 0}$ orthogonal with respect to the Sobolev inner product

$$\langle p,q\rangle_S=\int_{\mathbb{R}} p(x)q(x)d\mu_0+\lambda\int_{\mathbb{R}} p'(x)q'(x)d\mu_1,$$

with $\lambda>0$, $d\mu_0=e^{-x^2}dx$, $d\mu_1=\frac{x^2+a}{x^2+b}e^{-x^2}dx$, $a,b\in\mathbb{R}^+$ and $a\neq b$. It is well known that (μ_0,μ_1) is a pair of symmetric $(1,1)-$coherent measures. This means that there exist sequences $\{a_n\}_{n\in\mathbb{N}}$, $\{b_n\}_{n\in\mathbb{N}}$, $a_n\neq b_n$ for every $n\in\mathbb{N}$, such that the algebraic relation

$$H_n(x)+b_{n-2}H_{n-2}(x)=Q_n(x)+a_{n-2}Q_{n-2}(x), n\geq 2,$$

is satisfied, where $\{Q_n(x)\}_{n\geq 0}$ is the sequence of monic orthogonal polynomials associated with μ_1 and $\{H_n(x)\}_{n\geq 0}$ is the sequence of monic Hermite polynomials. We will study the relative asymptotics for Sobolev scaled polynomials and we will obtain Mehler-Heine type formulas, among others.

Keywords: Hermite (1,1)-coherent pairs, Sobolev Polynomials, Asymptotic properties
2010 MSC: 33C25; 42C05

1. Introduction

Diagonalized spectral methods using either generalized Laguerre functions (see [13],[14]) or Hermite functions (see [15]) have been recently used for boundary value problems associated with second order and fourth order differential equations of elliptic type in the positive real semi-axis and the real line, respectively. Taking into account the weak formulation of the boundary value problem such a functions are orthogonal/biorthogonal and constitute a complete basis with respect to the corresponding Sobolev inner product. Thus, Fourier-like Sobolev orthogonal basis functions are constructed for the diagonalized spectral method instead of the usual one based on the standard orthogonality. In such a way, optimal error estimates can be deduced and these approaches are competitive with the standard non diagonal spectral methods despite the fact the

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