Density of Polynomials in Sub-Bergman Hilbert Spaces

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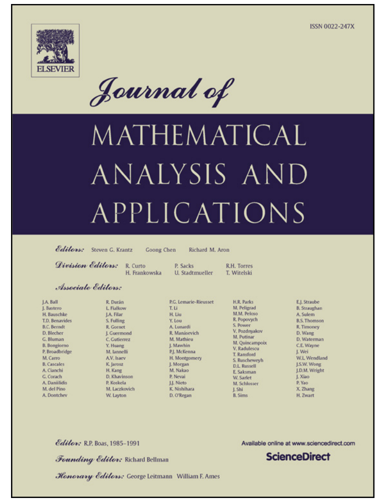

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Abstract. The sub-Bergman Hilbert spaces are analogues of de Branges-Rovnyak spaces in the Bergman space setting. We prove that the polynomials are dense in sub-Bergman Hilbert spaces. This answers the question posted by Zhu in the affirmative.

1. Introduction

Let A be a bounded operator on a Hilbert space H. We define the range space $\mathcal{M}(A) = AH$, and endow it with the inner product

$$\langle Af, Ag\rangle_{\mathcal{M}(A)} = \langle f, g\rangle_H, \qquad f, g \in H \ominus \mathrm{Ker}A.$$

Let $\mathbb{D}$ denote the unit disk. Let L^2 denote the Lebesgue space of square integrable functions on the unit circle $\partial\mathbb{D}$. The Hardy space H^2 is the subspace of analytic functions on $\mathbb{D}$ whose Taylor coefficients are square summable. Then it can also be identified with the subspace of L^2 of functions whose negative Fourier coefficients vanish. The Toeplitz operator on the Hardy space H^2 with symbol f in $L^\infty(\mathbb{D})$ is defined by

$$T_f(h) = P(fh),$$

for $h \in H^2(\mathbb{D})$. Here P be the orthogonal projections from L^2 to H^2.

Let b be a function in the closed unit ball of $H^\infty(\mathbb{D})$, the space of bounded analytic functions on the unit disk. The de Branges-Rovnyak space $\mathcal{H}(b)$ is defined to be the space $(I - T_b T_{\bar{b}})^{1/2} H^2$. We also define the space $\mathcal{H}(\bar{b})$ in the same way as $\mathcal{H}(b)$, but with the roles of b and $\bar{b}$ interchanged, i.e.

$$\mathcal{H}(\bar{b}) = (I - T_{\bar{b}} T_b)^{1/2} H^2.$$

The spaces $\mathcal{H}(b)$ and $\mathcal{H}(\bar{b})$ are also called sub-Hardy Hilbert spaces (the terminology comes from the title of Sarason's book [**4**]).

The Bergman space A^2 is the space of analytic functions on $\mathbb{D}$ that are square-integrable with respect to the normalized Lebesgue area measure dA. For $u \in$

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