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Smooth solutions to diffusion approximation radiation hydrodynamics equations

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A B S T R A C T

In this paper, we consider the smooth solutions to the Cauchy problem for the diffusion approximation radiation hydrodynamics equations in $\mathbb{R}^3$. The model describes the interaction of an inviscid gas with photons. The local existence and uniqueness of smooth solutions is obtained by using the energy method with more subtle energy estimates.

1. Introduction

The key aim of the radiation hydrodynamics is to include the radiation effects into hydrodynamics. The importance of thermal radiation in physical problems increases as the temperature is raised. At moderate temperatures (say, thousands of degrees Kelvin), the role of the radiation is transporting energy by radiative processes. At higher temperatures (say, millions of degrees Kelvin), the energy and momentum densities of the radiation field may become dominate the corresponding fluid quantities. In this case, the radiation field significantly affects the dynamics of the fluid. In this paper, we are interested diffusion approximation (also called the Eddington approximation) model in radiation hydrodynamics, which is the energy flow due to radiative process in a semi-quantitative sense, and is particularly accurate if the specific intensity of radiation is almost isotropic (cf. [15]). Such systems can be used to simulate, for instance, nonlinear stellar pulsation, supernova explosions and stellar winds in astrophysics and so on (cf. [10,14,17]). The diffusion approximation is valid for optically thick regions where the photons emitted by the gas have a high probability of reabsorption within the region (cf. [1]). Based on the standard hydrodynamics, the governing

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equations of the diffusion approximation in radiation hydrodynamics for 3-D flow of a inviscid gas, can be written in terms of Euler coordinates as follows:

$$\rho_t + div(\rho u) = 0, \tag{1.1}$$

$$\rho(u_t + u \cdot \nabla u) + \nabla P = -\nabla n, \tag{1.2}$$

$$c_\nu \rho(\theta_t + u \cdot \nabla\theta) + P divu = n - \theta^4 + u \cdot \nabla n, \tag{1.3}$$

$$n_t - \Delta n = \theta^4 - n. \tag{1.4}$$

Here, the unknowns are (ρ, u, θ, n), where $\rho = \rho(x,t) > 0, \theta = \theta(x,t), u = u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))$, for $t > 0$, $x \in \mathbb{R}^3$ denote the mass density, temperature and velocity field of the fluid respectively, and $n = n(x,t) > 0$ for $t > 0$, $x \in \mathbb{R}^3$ denote the radiation field, $P = R\rho\theta$ is the material pressure, $c_\nu > 0$ being the heat conductivity, $R = c_\nu(\gamma - 1)$, and $\gamma > 1$ being the specific heat ratio. More discuss of this system can be seen [1,5,15].

Due to its physical importance, complexity, rich phenomena, and mathematical challenges, there is large of literature on the studies of radiation hydrodynamics from the mathematical/physical point of view, see, for example [2,3,6,18]. Specially, for diffusion approximation model, in [7], Jiang et al. showed the global existence of smooth solutions for one-dimensional case with viscosity. However, the effect of radiation on the momentum (i.e. ∇n in (1.2)) is omitted for technical reason, and for full model, the global existence results can be seen [4]. The global well-posedness and large time behavior of classical solutions for multi-dimensional case see [5]. For inviscid case, Lin, Coulombel and Goudon consider a simplified model as follows (see [12]):

$$\begin{aligned}
&\rho_t + (\rho u)_x = 0,\\
&(\rho u)_t + (\rho u^2 + P)_x = 0,\\
&\left(\rho\theta + \frac{1}{2}\rho u^2\right)_t + \left(\left(\rho\theta + \frac{1}{2}\rho u^2\right)u + Pu\right)_x = n - \theta^4,\\
&-n_{xx} = \theta^4 - n,
\end{aligned} \tag{1.5}$$

which describes the interaction between an inviscid gas and photons. They showed the existence of smooth traveling waves, called "shock profiles", when the strength of shock is small. While, system (1.5) can be seen as a stationary radiation field case and excluded the effect of radiative pressure from the momentum equation of (1.1)–(1.4). As a deep simplified model of (1.5), the "radiating gases" model reads as follows:

$$\begin{aligned}
&u_t + (\frac{u^2}{2})_x = -q_x,\\
&-q_{xx} + q = -u_x.
\end{aligned} \tag{1.6}$$

The thorough study on (1.6) motivated a lot of works, see, for example, [8,9,11,16] and the references cited therein.

In this paper, we are interested in the local existence of smooth solutions to system (1.1)–(1.4) with the initial conditions:

$$(\rho, u, \theta, n)|_{t=0} = (\rho_0, u_0, \theta_0, n_0)(x), \quad x \in \mathbb{R}^3. \tag{1.7}$$

To obtain this result, we use an iteration and the Banach contraction mapping principle, a standard procedure see, e.g., [13]. However, we should point out here that the main difficulties in the proof lie in dealing with the nonlinear and non-local terms in (1.1)–(1.4). Especially for the nonlinear term θ^4 and radiation