



Global solutions in a high-dimensional two-species chemotaxis model with Lotka–Volterra competitive kinetics



Qingshan Zhang^{a,*}, Yuxiang Li^b

^a Department of Mathematics, Henan Institute of Science and Technology, Xinxiang 453003, PR China

^b Department of Mathematics, Southeast University, Nanjing 211189, PR China

ARTICLE INFO

Article history:

Received 30 August 2017

Available online 25 July 2018

Submitted by C. Gutierrez

Keywords:

Two-species chemotaxis system

Logistic source

Global solution

Weak solution

ABSTRACT

This paper deals with the two-species chemotaxis system with logistic source

$$\begin{cases} u_t = \Delta u - \chi_1 \nabla \cdot (u \nabla w) + \mu_1 u(1 - u - a_1 v), & x \in \Omega, t > 0, \\ v_t = \Delta v - \chi_2 \nabla \cdot (v \nabla w) + \mu_2 v(1 - a_2 u - v), & x \in \Omega, t > 0, \\ w_t = \Delta w - \lambda w + \alpha u + \beta v, & x \in \Omega, t > 0 \end{cases}$$

under homogeneous Neumann boundary condition in a smooth bounded domain $\Omega \subset \mathbb{R}^n$ ($n \geq 1$). It is proved that in convex domains the problem possesses a unique global bounded solution if μ_1 and μ_2 are large enough. Moreover, we establish the existence of global weak solution for any $\mu_1 > 0$ and $\mu_2 > 0$.

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

In this paper we consider global existence for the following two-species chemotaxis system with logistic source

$$\begin{cases} u_t = \Delta u - \chi_1 \nabla \cdot (u \nabla w) + \mu_1 u(1 - u - a_1 v), & x \in \Omega, t > 0, \\ v_t = \Delta v - \chi_2 \nabla \cdot (v \nabla w) + \mu_2 v(1 - a_2 u - v), & x \in \Omega, t > 0, \\ w_t = \Delta w - \lambda w + \alpha u + \beta v, & x \in \Omega, t > 0 \end{cases} \quad (1.1)$$

with homogeneous Neumann boundary conditions

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = 0, \quad x \in \partial \Omega, t > 0 \quad (1.2)$$

* Corresponding author.

E-mail address: qingshan11@yeah.net (Q. Zhang).

and initial conditions

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \quad w(x, 0) = w_0(x), \quad x \in \Omega \quad (1.3)$$

in a smooth bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 1$, where χ_1 , χ_2 , μ_1 , μ_2 , a_1 , a_2 , λ , α and β are all positive constants.

In mathematical biology, system (1.1) is used to describe the movement of two competitive biological species in response to the same chemical stimulus secreted by themselves. The mutual competition between them takes place on account of the classical Lotka–Volterra kinetics. The unknown functions $u(x, t)$ and $v(x, t)$ represent the population densities of two competitive biological species, respectively. The concentration of the common chemical substance is denoted by $w(x, t)$. The parameters χ_1 and χ_2 are chemotactic sensitivities; μ_1 and μ_2 denote the growth rates of two species; a_1 and a_2 measure the strength of the competition between u and v . The size of α , β and λ describe the production and decay of the chemical signal.

In order to understand the system (1.1) better, let us mention the classical single-species chemotaxis model [17]

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla w), & x \in \Omega, \quad t > 0, \\ w_t = \Delta w - w + u, & x \in \Omega, \quad t > 0, \end{cases} \quad (1.4)$$

which has been studied in different contexts (see the surveys [2,11,14] and the references therein). From a mathematical point of view, one of key issues is whether the solutions of (1.4) are global existence or blow up. It has been shown finite-time blow-up occurs either when $n = 2$ and $\int_{\Omega} u_0$ is larger than the threshold number [9,15,24], or $n \geq 3$ for any prescribed value of $\int_{\Omega} u_0$ [24,34]. Since the phenomenon of blow-up is uttermost in the real biological systems, a large number of variations of the original model (1.4) have been developed to guarantee global existence of bounded solutions [11]. Among the variations, adding the logistic kinetics term $\kappa u - \mu u^2$ to the first equation in (1.4) is an effective way to prevent the possibility of explosion. It is proved that blow-up is ruled out completely either $n \leq 2$ [27,28], or $n \geq 3$ with suitably large μ [33,35]. Moreover, eventual smooth global weak solution has been constructed in three dimension [18].

System (1.1) is a generalization of the classical Keller–Segel model (1.4) to the two-species and signal-stimuli case. Recently, it has been considered by several authors. Without logistic source in the two-species chemotaxis, the system inherits some properties from the classical model (1.4). Global existence versus blow-up of the solutions has been investigated in [3,7,8]. The global existence and asymptotic behavior of solution is also studied for the two-species chemotaxis model with slow chemical diffusion [25] and for the two-dimensional Cauchy problem [37].

While the diffusion of chemical signal is much faster than the two populations, system (1.1) can be written as

$$\begin{cases} u_t = \Delta u - \chi_1 \nabla \cdot (u \nabla w) + \mu_1 u(1 - u - a_1 v), & x \in \Omega, \quad t > 0, \\ v_t = \Delta v - \chi_2 \nabla \cdot (v \nabla w) + \mu_2 v(1 - a_2 u - v), & x \in \Omega, \quad t > 0, \\ 0 = \Delta w - \lambda w + \alpha u + \beta v, & x \in \Omega, \quad t > 0. \end{cases}$$

With strong logistic damping effect, the model allows for global bounded solution [4,19,22,29,30]. Moreover, conditions of coexistence in the case $a_1, a_2 \in [0, 1]$ [4,22,30] and competitive exclusion when $a_1 > 1 > a_2 > 0$ [22,29] are established, and the convergence rates for solutions of the above system have also been studied in [22].

Concerning the fully system (1.1) considered in the present paper, it is shown in [1] if $n \leq 2$ the corresponding Neumann problem possesses a unique global bounded solution for all positive parameters. It is also

Download English Version:

<https://daneshyari.com/en/article/8899356>

Download Persian Version:

<https://daneshyari.com/article/8899356>

[Daneshyari.com](https://daneshyari.com)