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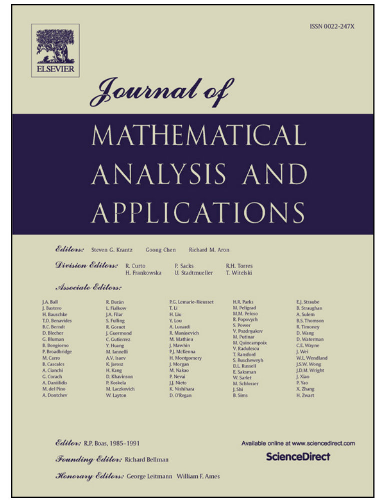

Liouville type theorem for system of elliptic differential inequalities on Riemannian manifolds

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LIOUVILLE TYPE THEOREM FOR SYSTEM OF ELLIPTIC DIFFERENTIAL INEQUALITIES ON RIEMANNIAN MANIFOLDS

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ABSTRACT. In this paper we deal with the uniqueness of nonnegative solutions to system of quasilinear elliptic differential inequalities on Riemannian manifolds, and we obtain the Liouville type theorem only in terms of the volume growth of geodesic ball. By applying the perturbation and gluing techniques, a counterexample is constructed to show the sharpness of the volume growth condition.

1. INTRODUCTION

In this paper, we are mainly focused on the uniqueness of nonnegative solutions to the following system of elliptic differential inequalities

$$\begin{cases} \Delta_m u + v^{\sigma_1} \le 0, \\ \Delta_n v + u^{\sigma_2} \le 0. \end{cases} \tag{1.1}$$

on a geodesically complete connected noncompact Riemannian manifold M. Here $\sigma_1 > n-1, \sigma_2 > m-1, m > 1, n > 1$ are given exponents, and $\Delta_m u := \operatorname{div}(|\nabla u|^{m-2}\nabla u)$ and $\Delta_n v := \operatorname{div}(|\nabla v|^{n-2}\nabla v)$.

The problem concerning the uniqueness of nonnegative solution has been extensively studied in many spaces, especially in the Euclidean space, see [3] [4] [5] [6] [7] [8] [9] [13] [14] [16] [17] [18] [19] [20]. It originated from the celebrated work [9] of Gidas and Spruck. They considered the equation

$$\Delta u + u^\sigma = 0 \quad \text{in } \mathbb{R}^N, \tag{1.2}$$

and proved that if

$$1 < \sigma < \frac{N+2}{N-2},$$

then the only non-negative solution of (1.2) is zero. On the contrary, if $\sigma \ge \frac{N+2}{N-2}$, then there exists a positive solution. For example, in the case $\sigma = \frac{N+2}{N-2}$ it is

$$u(x) = \frac{c_N}{(1+|x|^2)^{\frac{N-2}{2}}},$$

with some $c_N > 0$. Hence, the critical value of the exponent σ for the problem (1.2) is $\frac{N+2}{N-2}$.

Recall the Lane-Emden system in $\mathbb{R}^N$ with $N \ge 3$, which is an equation version of (1.1) when $m = n = 2$

$$\begin{cases} \Delta u + v^{\sigma_1} = 0, \\ \Delta v + u^{\sigma_2} = 0. \end{cases} \tag{1.3}$$

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