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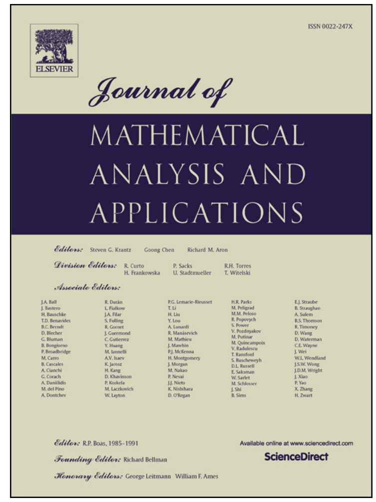

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REGULARITY PROPERTIES OF THE HEAT KERNEL AND AREA INTEGRAL CHARACTERIZATION OF HARDY SPACE $H^1_{\mathcal{L}}$ RELATED TO DEGENERATE SCHRÖDINGER OPERATORS

JIZHENG HUANG, PENGTAO LI*, AND YU LIU

Abstract. In this paper we introduce the area integral associated with the heat semigroup $\{T_t\}_{t>0}$ generated by a degenerate Schrödinger operator $\mathcal{L}$. We obtain a regularity estimate of the kernels of the semigroup $\{T_t\}_{t>0}$. This regularity together with a reproducing formula enables us to establish the area integral characterization of the Hardy space associated with $\mathcal{L}$.

Contents

1. Introduction

It is a well-established fact that, for the purposes of the studies on harmonic analysis or partial differential equations, the right substitute for $L^p(\mathbb{R}^n)$ with $p \in (0,1]$, is the (real) Hardy space $H^p(\mathbb{R}^n)$, or it's local version $h^p(\mathbb{R}^n)$. Denote by Δ the Laplace operator on $\mathbb{R}^n$. Then $H^1(\mathbb{R}^n)$ can be characterized by the heat maximal function $\sup_{t>0}|e^{-t\Delta}f(x)|$ (cf. Stein [15]). In a sense, $H^1(\mathbb{R}^n)$ can be seen as a class of Hardy spaces associated with the operator $-\Delta$. Let $\mathcal{L}$ denote a general differential operator, such as second order elliptic self-adjoint operators in divergence form, degenerate Schrödinger operators with nonnegative potential, Schrödinger operators with nonnegative potential and so on. In recent years, the Hardy spaces associated with $\mathcal{L}$ become one of hot issues of harmonic analysis. We refer to [5, 4, 3, 10, 1, 11, 12, 14, 18] and the references therein. In particular, [4] and [18] deal with the Hardy spaces associated with the degenerate Schrödinger operators.

In [7], Fefferman and Stein established the area integral characterization of the Hardy spaces $H^p(\mathbb{R}^n)$. From then on, such characterization was extended to other settings. We refer the reader to [3, 2, 10] for further information. Let $\mathcal{L}$ be a degenerate Schrödinger operator on $\mathbb{R}^n$, which is defined as

$$\mathcal{L}f(x) =: -\frac{1}{\omega(x)}\sum_{i,j}\partial_i(a_{ij}(\cdot)\partial_j f)(x) + V(x)f(x),$$

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