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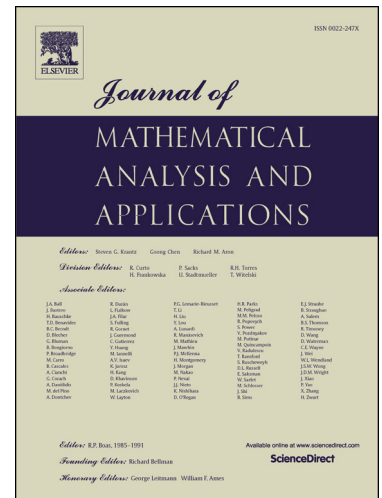
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TOEPLITZ OPERATORS ON HARMONICALLY WEIGHTED BERGMAN SPACES AND APPLICATIONS TO COMPOSITION OPERATORS ON DIRICHLET SPACES

OMAR EL-FALLAH, HOUSAME MAHZOULI, IBRAHIM MARRHICH, HATIM NAQOS

ABSTRACT. In this paper we consider Toeplitz operators acting on Bergman spaces associated with harmonic weights. We give a complete description of those operators which belong to p -Schatten classes. We also obtain an upper estimate for the reproducing kernel of these spaces. Applications to composition operators on Dirichlet spaces are given.

1. INTRODUCTION

Let \mathbb{D} denote the open unit disc of the complex plane \mathbb{C} and let $dA(z) = r dr d\theta / \pi$ denote the normalized Lebesgue area measure on \mathbb{D} . Let $\omega : \mathbb{D} \rightarrow]0, +\infty[$ be an integrable weight on \mathbb{D} which is bounded below by a positive constant on each compact set of \mathbb{D} . The weighted Bergman space associated with ω is given by

$$A_{\omega}^2 = \{f \in \text{Hol}(\mathbb{D}) : \|f\|_{\omega} = \left(\int_{\mathbb{D}} |f(z)|^2 \omega(z) dA(z) \right)^{\frac{1}{2}} < \infty\}.$$

The standard Bergman spaces on \mathbb{D} , denoted A_{α}^2 , correspond to $\omega(z) = (1 + \alpha)(1 - |z|^2)^{\alpha}$. By an easy and standard argument, one can see that the evaluation functionals at points of \mathbb{D} are bounded on A_{ω}^2 . So, A_{ω}^2 has a reproducing kernel denoted by K^{ω} (or simply K).

The Toeplitz operator T_{μ} acting on A_{ω}^2 associated with the positive Borel measure μ is given by

$$T_{\mu}f(z) := \int_{\mathbb{D}} K(z, \zeta) f(\zeta) \omega(\zeta) d\mu(\zeta), \quad (z \in \mathbb{D}).$$

A description of measures μ such that T_{μ} is bounded, compact or membership to p -Schatten classes S_p have been obtained for several weighted Bergman spaces by several authors. For standard Bergman spaces D . Luecking gave an explicit geometric characterization on a measure μ ensuring that $T_{\mu} \in S_p$, for all $p > 0$. Thereafter, P. Lin and R. Rochberg [14] consider another class of weights, called Oleinik -Perel'man weights, and proved a similar result for $p \geq 1$. In 2013, J. Pau in [16] extends this characterization for all $p \in (0, 1)$.

In this paper we are interested in the case where ω is harmonic. This case is motivated by the study of composition operators on Dirichlet spaces (see section 5).

Note that, for a positive harmonic weight ω on \mathbb{D} , there exists a unique positive Borel measure ν on the unit circle \mathbb{T} such that $\omega = P_{\nu}$, where

$$P_{\nu}(z) = \int_{\mathbb{T}} \frac{1 - |z|^2}{|\zeta - z|^2} d\nu(\zeta),$$

is the Poisson transform of ν . In this case we will adopt the following notations $A_{\omega}^2 = A_{\nu}^2$, $\|\cdot\|_{\omega} = \|\cdot\|_{\nu}$ and $K^{\omega} = K^{\nu}$.

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