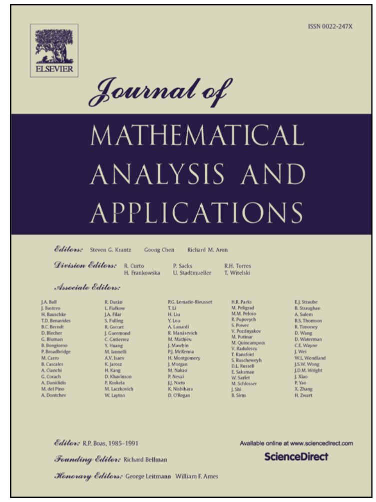
Accepted Manuscript



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PII: S0022-247X(18)30496-7
DOI: https://doi.org/10.1016/j.jmaa.2018.06.005
Reference: YJMAA 22321

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 25 September 2017

A MULTIPLICITY RESULT FOR A NONLINEAR FRACTIONAL SCHRÖDINGER EQUATION IN $\mathbb{R}^N$ WITHOUT THE AMBROSETTI-RABINOWITZ CONDITION

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Abstract. In this paper we study the existence and the multiplicity of positive solutions for the following class of fractional Schrödinger equations

$$\epsilon^{2s}(-\Delta)^s u + V(x)u = f(u) \text{ in } \mathbb{R}^N,$$

where $\epsilon > 0$ is a parameter, $s \in (0,1)$, $N > 2s$, $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous positive potential, and $f : \mathbb{R} \to \mathbb{R}$ is a $\mathcal{C}^1$ superlinear nonlinearity which does not satisfy the Ambrosetti-Rabinowitz condition. The main result is established by using minimax methods and Ljusternik-Schnirelmann theory of critical points.

1. Introduction

Recently a great attention has been devoted to the study of the standing wave solutions for the following nonlinear fractional Schrödinger equation:

$$i\epsilon\frac{\partial\Psi}{\partial t} = \epsilon^{2s}(-\Delta)^s\Psi + (V(z)+E)\Psi - g(|\Psi|)\Psi \text{ for all } z \in \mathbb{R}^N, \tag{NLS}$$

where $N > 2s$, $s \in (0,1)$, $\epsilon > 0$, $E \in \mathbb{R}$, and V, g are continuous functions. This equation plays a fundamental role in fractional quantum mechanics and it has been proposed by Laskin in [33, 34]. By a standing wave solution we mean a function of the type $\Psi(z,t) = exp(-iEt/\epsilon)u(z)$. Then, $\Psi(z,t)$ satisfies (NLS) if and only if u is a solution of the following fractional elliptic problem:

$$\begin{cases} \epsilon^{2s}(-\Delta)^s u + V(z)u = f(u) \text{ in } \mathbb{R}^N, \\ u \in H^s(\mathbb{R}^N), \quad u > 0 \text{ on } \mathbb{R}^N. \end{cases} \tag{P_ϵ}$$

Here, the fractional Laplacian operator $(-\Delta)^s$ is defined for a function $u : \mathbb{R}^N \to \mathbb{R}$ belonging to the Schwartz class by

$$\mathcal{F}((-\Delta)^s u)(\xi) = |\xi|^{2s}\mathcal{F}(u)(\xi), \; \xi \in \mathbb{R}^N,$$

where $\mathcal{F}$ denotes the Fourier transform, that is,

$$\mathcal{F}(u)(\xi) = \frac{1}{(2\pi)^{\frac{N}{2}}}\int_{\mathbb{R}^N} e^{-i\xi\cdot x}u(x)\, dx \equiv \widehat{u}(\xi).$$

Also $(-\Delta)^s u$ can be equivalently represented [26, Lemma 3.2] as

$$(-\Delta)^s u(x) = -\frac{1}{2}C(N,s)\int_{\mathbb{R}^N}\frac{(u(x+y)+u(x-y)-2u(x))}{|y|^{N+2s}}\, dy, \; \forall x \in \mathbb{R}^N,$$

where

$$C(N,s) = \left(\int_{\mathbb{R}^N}\frac{(1-cos\xi_1)}{|\xi|^{N+2s}}d\xi\right)^{-1}, \; \xi = (\xi_1, \xi_2, \ldots, \xi_N).$$

2010 *Mathematics Subject Classification.* 47G20, 35R11, 35A15, 58E05.
Key words and phrases. Fractional Schrödinger equation; Nehari manifold; Ljusternik-Schnirelmann theory.