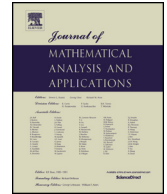




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Well-posedness and blow-up phenomena for an integrable three-component Camassa–Holm system [☆]

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ABSTRACT

This paper studies the Cauchy problem for an integrable three-component Camassa–Holm system. We first establish the local well-posedness with initial condition in Besov spaces. Then we prove a blow-up criteria by arguing inductively with respect to the regularity index. Moreover, we derive a Riccati-type differential inequality by using the structure of equations, and also prove a new blow-up criteria with sufficient conditions on initial condition, whose proof is based on the conservative property of potential densities along the characteristic.

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1. Introduction

In this paper we consider the Cauchy problem for the following integrable three-component Camassa–Holm system with cubic nonlinearity:

$$\begin{cases} m_{1t} + u_2 g m_{1x} - m_3 (u_{2x} f - u_2 g) - m_1 (3u_2 f - m_3 u_2) = 0, \\ m_{2t} + u_2 g m_{2x} + m_2 (3u_{2x} g + m_3 u_2) = 0, \\ m_{3t} + u_2 g m_{3x} - m_3 (2u_2 f + u_{2x} g - m_3 u_2) = 0, \\ m_i|_{t=0} = m_{i0}, \quad m_2|_{t=0} = m_{20}, \quad m_3|_{t=0} = m_{30}, \end{cases} \quad (1.1)$$

where

$$m_i = u_i - u_{ixx}, \quad i = 1, 2, 3, \quad f = u_3 - u_{1x}, \quad g = u_1 - u_{3x}.$$

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The system (1.1) was proposed by Li, Liu and Popowicz in [26], which is associated with the following 3×3 matrix spectral problem

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}_x = \begin{pmatrix} 0 & 0 & 1 \\ \lambda m_1 & 0 & \lambda m_3 \\ 1 & \lambda m_2 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{1.2}$$

where m_1, m_2, m_3 are three potentials and λ is a constant spectral parameter. Meanwhile, the authors pointed out that the system (1.1) can be rewritten as a bi-Hamiltonian system by

$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}_t = \mathcal{L}_1 \begin{pmatrix} \frac{\delta H_0}{\delta m_1} \\ \frac{\delta H_0}{\delta m_2} \\ \frac{\delta H_0}{\delta m_3} \end{pmatrix} = \mathcal{L}_2 \begin{pmatrix} \frac{\delta H_1}{\delta m_1} \\ \frac{\delta H_1}{\delta m_2} \\ \frac{\delta H_1}{\delta m_3} \end{pmatrix}, \tag{1.3}$$

with

$$H_0 = \int (u_1 f + u_{2x} g) g m_2 dx, \quad H_1 = \int g m_2 dx,$$

where \mathcal{L}_1 and \mathcal{L}_2 denote the Hamiltonian operators in terms of variables m_1, m_2, m_3 , see [26] for more details. Moreover, they have suggested the way to construct infinitely many conserved quantities for the integrable system (1.1).

If $u_3 \equiv 0$, the system (1.1) reduces to the two-component Novikov system

$$\begin{cases} m_t + uvm_x + 3vu_xm = 0, \\ n_t + uvn_x + 3uv_xn = 0, \end{cases} \tag{1.4}$$

where $m = u - u_{xx}$, $n = v - v_{xx}$. The system (1.4) was proposed by Geng and Xue [12], in which they calculated the N-peakons and conserved quantities and found a Hamiltonian structure. The associated bi-Hamiltonian structure for (1.4) was presented in [24]. It is proved that the Geng–Xue system also admits shockpeakons [28], which was first introduced for the Degasperis–Procesi equation [27]. Notice that if $u = v$ in (1.4), we obtain the famous Novikov equation [31]

$$m_t + 3uu_xm + u^2m_x = 0, \quad m = u - u_{xx}, \tag{1.5}$$

which possesses a bi-Hamiltonian structure, an infinite sequence of conserved quantities and the explicit formulas for multipeakon solutions [20,21]. In the years immediately following, many attempts have been made to the studying of well-posedness, blow-up phenomena and analyticity for (1.5) in Sobolev spaces and Besov spaces, see for example [14,19,22,30,38].

By taking $u_2 \equiv 1$ and $u_3 \equiv 0$ in (1.1), it becomes the Degasperis–Procesi (DP) equation

$$m_t + um_x + 3u_xm = 0, \quad m = u - u_{xx}. \tag{1.6}$$

The formal integrability of DP equation can be proved by constructing a Lax pair, while the direct and inverse scattering approach to pursue it can be found in [8]. The DP equation has bi-Hamiltonian structure [8] and an infinite sequence of conserved quantities, and admits exact peakon solutions which are analogous to the Camassa–Holm peakons [2,6,7]. It is worth pointing out that solutions of this type are not mere abstractizations: the peakons replicate a feature that is characteristic for the waves of great height—waves of largest amplitude that are exact solutions of the governing equations for irrotational water waves [3,33]. Recently, by applying the dressing method to smooth localized solutions of (1.6), Constantin and Ivanov

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