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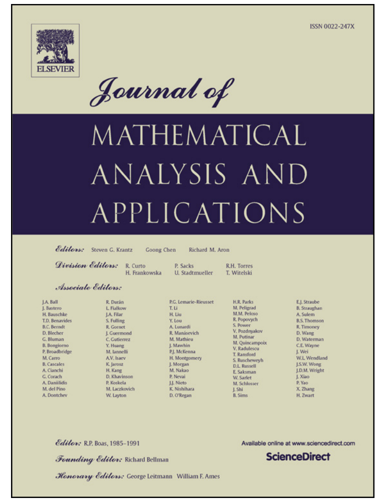

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ON A POLYNOMIAL SCALAR PERTURBATION OF A SCHRÖDINGER SYSTEM IN L^p-SPACES

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Dedicated to Prof. H. Bouslous
on the occasion of his 65^{th} *birthday*

Abstract. In the paper [10] the L^p-realization L_p of the matrix Schrödinger operator $\mathcal{L}u = div(Q\nabla u) + Vu$ was studied. The generation of a semigroup in $L^p(\mathbb{R}^d, \mathbb{C}^m)$ and characterization of the domain $D(L_p)$ has been established. In this paper we perturb the operator L_p of by a scalar potential belonging to a class including all polynomials and show that still we have a strongly continuous semigroup on $L^p(\mathbb{R}^d, \mathbb{C}^m)$ with domain embedded in $W^{2,p}(\mathbb{R}^d, \mathbb{C}^m)$. We also study the analyticity, compactness, positivity and ultracontractivity of the semigroup and prove Gaussian kernel estimates. Further kernel estimates and asymptotic behaviour of eigenvalues of the matrix Schrödinger operator are investigated.

1. Introduction

While the scalar theory of second-order elliptic operators with unbounded coefficients is by now well developed (cf. [11] and the references therein), there is still few research works, at least in the framework of semigroup theory, for systems of parabolic equations with unbounded coefficients. To our knowledge one of the first papers dealing with such kind of systems is [7]. Subsequently, there were some other publications [1, 2, 4]. Here the strategy in these references is quite different from that in [7]. Namely, in [1, 2, 4] solutions to the parabolic equation are at first constructed in the space of bounded and continuous functions. Afterwards the semigroup is extrapolated to the L^p-scale. This approach cannot give precise information about the domain of the generator of the semigroup.

Recently in [10] a noncommutative Dore–Venni theorem due to S. Monniaux and J. Prüss [16] was used to obtain generation of C_0-semigroups for matrix Schrödinger operators of type $\mathcal{L} = div(Q\nabla\cdot) + V$ in L^p-spaces, where V is a matrix potential whose entries can grow like $|x|^r$ for some $r \in [1, 2)$. This approach permits to obtain the maximal inequality

$$(1.1) \qquad \|div(Q\nabla u)\|_{L^p(\mathbb{R}^d,\mathbb{C}^m)} + \|Vu\|_{L^p(\mathbb{R}^d,\mathbb{C}^m)} \le C\|div(Q\nabla u) + Vu\|_{L^p(\mathbb{R}^d,\mathbb{C}^m)}$$

for all $u \in C_c^\infty(\mathbb{R}^d, \mathbb{C}^m)$ and some positive constant C independent of u.

An other approach is to use form methods and Beurling-Denny criterion to prove generation of C_0-semigroup in L^p-spaces. This approach works for symmetric matrix Schrödinger operators, but no information about the domain of the generator can be obtained, see the recent paper [13].

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