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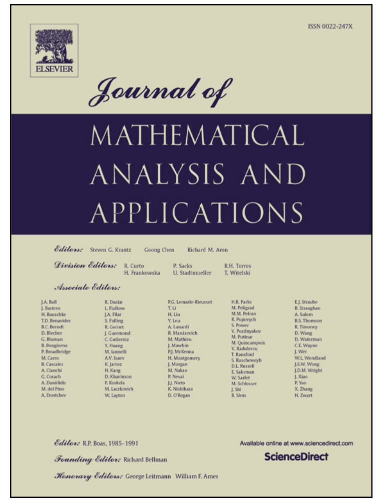

Examples of Generalized Bi-Circular Idempotents on Spaces of Continuously Differentiable Functions

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EXAMPLES OF GENERALIZED BI-CIRCULAR IDEMPOTENTS ON SPACES OF CONTINUOUSLY DIFFERENTIABLE FUNCTIONS

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Abstract. In this note we investigate the form of a class of idempotents, called generalized bi-circular idempotents on spaces of continuously differentiable functions defined on the interval [0, 1] and with values in the complex numbers.

1. Introduction

The Mazur-Ulam theorem formulates that some translation of a surjective and distance preserving map on a complex normed space is real linear. In this note we consider a class of idempotent maps, defined on normed spaces of complex valued differentiable functions, associated with a surjective isometry. More precisely, we study idempotents with the property that the complex hull defined by one such idempotent and its complementary must contain a surjective isometry. The definition introduced here follows closely the definition of generalized bi-circular projection, however the maps involved are not assumed to be linear.

It is standard that "projection" refers to a bounded and idempotent linear operator. Important classes of projections include the contractive projections, or norm-one projections. These projections emerged as natural generalizations of orthogonal projections on a Hilbert space to a Banach space setting. Furthermore, 1-complemented subspaces of a Banach space are those given as the range of a contractive projection. Existence of 1-complemented subspaces has been a topic of research interest, see [13] and references therein.

Among the basic problems in Banach space theory, we mention the characterization of a space based on the structure of its projections and their admissible ranges. Various types of projections were studied by several researchers, as for example in [9], and [10]. In 2004, a class of projections, called bi-circular projections, were introduced by Stachó and Zalar in [14, 15]. A projection P is called bi-circular if $e^{i\alpha}P + e^{i\beta}(I - P)$ is a linear isometry for all $\alpha,\ \beta \in \mathbb{R}$. In [11], these projections were shown to be norm hermitian. A very interesting fact about these projections is that they characterize Hilbert spaces among JB*-triples, for details see [14]. Later, Fošner, Ilišević, and Li, in [6], considered

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