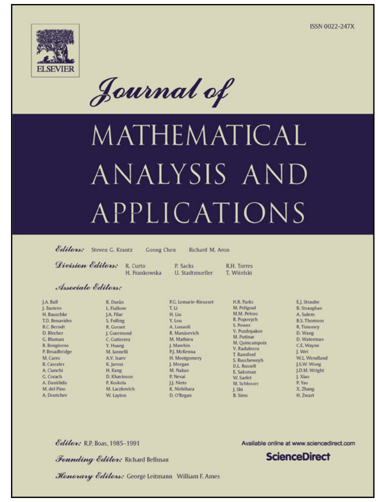

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Iz-iddine EL-Fassi

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ON THE GENERAL SOLUTION AND HYPERSTABILITY OF THE GENERAL RADICAL QUINTIC FUNCTIONAL EQUATION IN QUASI-β-BANACH SPACES

IZ-IDDINE EL-FASSI*[1]

ABSTRACT. The goal of this paper is to study the general solution of the following general radical quintic functional equation

$$f\left(\sqrt[5]{ax^5+by^5}\right)=rf(x)+sf(y)$$

for f a mapping from the field of real numbers into a vector space, where a, b, r, s are fixed nonzero reals. Also, we prove the generalized hyperstability results for the general radical quintic functional equation by using the fixed point theorem (cf. Dung and Hang in J. Math. Anal. Appl. 462 (2018) 131-147, Theorem 2.1) in quasi-β-Banach spaces. Namely, we show, under some weak natural assumptions, functions satisfying the above equation approximately (in some sense) must be actually solutions to it.

1. Introduction

One of the most interesting questions in the theory of functional analysis concerning the Ulam stability problem of functional equations is as follows: *Under what conditions a mathematical object satisfying a certain property approximately must be close to an object satisfying the property exactly ?*

Let us recall that the first stability problem concerning group homomorphisms was raised by Ulam [35] in 1940. We are given a group G and a metric group G' with metric $d(\cdot,\cdot)$. *Given* $\epsilon>0$, *does there exist* $\delta>0$ *such that if a function* $h: G\to G'$ *satisfies the inequality* $d(h(xy), h(x)h(y))\leqslant\delta$ *for all* $x, y\in G$, *then there is a homomorphism* $H: G\to G'$ *with* $d(h(x), H(x))\leqslant\epsilon$ *for all* $x\in G$ *?* In other words, we are looking for situations when the homomorphisms are stable; that is, if a mapping is almost a homomorphism, then there exists a true homomorphism near it. If the answer is affirmative, we would say that the equation is stable. In 1941, Hyers [25] considered the case of approximately additive mappings in Banach spaces and satisfying the well-known weak Hyers inequality controlled by a positive constant. The famous Hyers stability result that appeared in [25] was generalized by Aoki [3] and next by Rassias [31]. For the last decades, stability problems of various functional equations have been extensively investigated and generalized by many mathematicians [2, 5, 8, 11, 13, 22, 32, 33, 34]. Recently, the stability problem for the

*Corresponding author.

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