



Analyticity to transmission problem with delay in porous-elasticity<sup>☆</sup>Carlos A. Raposo<sup>a</sup>, Tijani A. Apalara<sup>b,\*</sup>, Joilson O. Ribeiro<sup>c</sup><sup>a</sup>Mathematics Department, Federal University of São João del-Rei, Brazil<sup>b</sup>University of Hafr Al Batin (UoHB), Saudi Arabia<sup>c</sup>Mathematics Department, Federal University of Bahia, Brazil**Abstract**

In this paper, we consider a one-dimensional transmission problem in a bounded domain with a delay in porous-elasticity. Using a semigroup theorem, under suitable assumption on the weight of the delay term, we establish the well-posedness of the system. Furthermore, using the method developed by Z. Liu and S. Zheng we show that the semigroup associated with the dissipative system is analytic and consequently exponentially stable.

*Keywords:* Analyticity, well-posedness, porous-elasticity, transmission problem, delay

**1. Introduction**

In this present work, we are concerned with the following transmission problem

$$\begin{aligned}
 u_{tt} - u_{xx} - b\varphi_x + \mu_1 u_t(x, t) + \mu_2 u_t(x, t - \tau) &= 0, & (x, t) \in \Omega \times (0, +\infty), \\
 \varphi_{tt} - \varphi_{xx} + bu_x + a\varphi + \xi\varphi_t &= 0, & (x, t) \in \Omega \times (0, +\infty), \\
 v_{tt} - v_{xx} - \beta\phi_x &= 0, & (x, t) \in (L_1, L_2) \times (0, +\infty), \\
 \phi_{tt} - \phi_{xx} + \beta v_x + \alpha\phi + \xi\phi_t &= 0, & (x, t) \in (L_1, L_2) \times (0, +\infty),
 \end{aligned} \tag{1}$$

where  $0 < L_1 < L_2 < L$ ,  $\Omega = ]0, L_1[ \cup ]L_2, L[$ ,  $\mu_1 > \mu_2$  are positive constants,  $\tau > 0$  is the delay, and  $a > b^2$ ,  $\alpha > \beta^2$ . System (1) is a porous system with localized delay effect. The system is such that a delay term acts in one part while the other part is indifferent to the effect of the delay. The resulting mathematical model is called transmission problem. Transmission problems arise frequently in several applications of physics and biology. We mention some results in this direction.

Dautray and Lions [1] discussed the linear transmission problem for hyperbolic equations and proved the existence and regularity of solutions. Raposo et al [2] considered a transmission problem for the Timoshenko system in which a part of the beam has friction and the other is purely elastic. They showed, using the energy method, that the dissipation produced by the frictional part is strong enough to produce exponential decay of the solution, no matter how small is its size. Similar result was earlier obtained by Rivera and Oquendo [3]. Alves et al [4] studied a transmission problem with localized Kelvin-Voigt viscoelastic damping and proved that corresponding semigroup is not exponentially stable, but the solution of the system decays polynomially (optimally) to zero as  $\frac{1}{t^2}$  when the initial data are taken over the domain. Recently, Wang et al [5] considered an interesting transmission model with two kinds of thermoelastic components; classical thermoelasticity and nonclassical thermoelasticity without dissipation. The two components were coupled at the interface satisfying certain transmission condition. They proved that the system lacks exponential decay rate but only sharp polynomial decay rate could be obtained. Concerning transmission problem in thermoelasticity of type III, Messaoudi and B. Said-Houari [6] proved that the thermal effect is strong enough to produce an exponential stability of the solution,

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