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Existence and uniqueness theorems for periodic Markov process and applications to stochastic functional differential equations

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ABSTRACT

This work is devoted to an $\mathbb{M}$-valued Markov process with a periodic transition probability function, where $\mathbb{M}$ is a Polish space. Sufficient conditions for the existence and uniqueness of the periodic Markov process are given. These existence theorems are rather general and the corresponding results in [1,11,15,23] are improved and generalized. The uniqueness theorems for the periodic Markov process are original. As applications, we study the existence, uniqueness and global attractivity of periodic solution for the periodic stochastic n-species Lotka–Voltrra competitive model with bounded delays and the periodic stochastic neural networks with infinite delay, respectively. Numerical simulations are presented to illustrate the main results.

1. Introduction

After the pioneering work of Itô [10], many scholars started to have considerable interest in the theory of stochastic differential equations (SDE) and made an extensive development on this issue (see [2,5,7,8,14,17,25,26]). These equations not only have richer dynamic properties than the corresponding differential equations without stochastic perturbation but also represent a more natural framework for mathematical modeling in many branches of the natural sciences (see [18,21]). Now the qualitative theories of SDE have been well developed (see [11,16]). However, the direction of the periodically SDE has not been developed very well. Till now only a few results have been obtained on this topic (see [7,8,11]). The authors in [7,8,11] gave some basic results for the existence of periodic solution of periodically SDE. But, these results are not suitable to the existence of periodic solution of general stochastic functional differential equations (SFDE), such as the stochastic retarded differential difference equations and the SFDE with infinity delay.

On the other hand, the time delays, due to various fields in science and engineering they can be found in, have received considerable attention. In the context of dynamical systems, a class of such systems is called

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functional differential equations (FDE), and many researchers have focused on the dynamic properties of the FDE (see [4,6,9,28]). Further, the existence of periodic solutions of periodically FDE also has been studied extensively (see [4,6,9,28]). Yoshizawa [28] shows that the periodically FDE has a periodic solution if its solutions are uniformly bounded and uniformly ultimately bounded. J.K. Hale [6] proves that if the solutions of a periodically FDE are uniformly bounded and point dissipative, then the periodically FDE has a periodic solution. The periodically FDE with infinite delay is studied by Hino and his cooperators [9].

However, as we know, there are few scholars focus on the periodically SFDE. Xu and his cooperators [23] studied the existence theorems for periodic $C([-\tau,0],\mathbb{R}^n)$-valued Markov process and applied the theorems to a class of periodic SFDE. They obtained that if the solutions of periodic SFDE are uniformly bounded, point dissipative and uniformly stochastically continuous, then there exists a periodic solution of the equations. Then Li and Xu [15] obtained the existence of a periodic solution of the periodic $BC((-\infty,0],\mathbb{R}^n)$-valued Markov process and applied the theories to the SFDE with infinite delay. To our knowledge, the proofs of Lemma 2.1 in [15] and Lemma 2.3 in [23] are based on the Theorem 3.2 in [8]. However, it is important to point out that Theorems 3.1 and 3.2 in [8] are almost entirely the same in their proofs, in which the Feller property of the time-homogeneous transition probability function of the Markov process is necessary. But the similar property for the periodic transition probability function of the Markov process is missed by Theorem 3.2 in [8] (We will elucidate it in the Appendix). What deserves special mention is that the proof of Theorem 3.2 in [8] is not suitable for the space $BC((-\infty,0],\mathbb{R}^n)$ since it is not a separable space. Therefore, the proof of the Lemma 2.1 in [15] is insufficient when $\tau=\infty$. Further, the main results in [15,23] focused only on the existence of the periodic solution, leaving the uniqueness of the periodic solution unattended. Motivated by the above discussion, it is interesting and meaningful to improve and generalize the results in [15,23]. In this paper, we will generalize the existence results for the periodic Markov process to an $\mathbb{M}$-valued Markov process with periodic transition probability function, where $\mathbb{M}$ is a Polish space. Then the uniqueness of the periodic Markov process is studied. As applications, we study the existence, uniqueness and global attractive of periodic solution of the periodic stochastic Lotka–Voltra competitive model with bounded delays and the periodic stochastic neural networks with infinite delay, respectively.

The rest of the paper is arranged as follows. Section 2 provides necessary notation and establishes the existence and uniqueness of the periodic Markov process. In Section 3, the theorems in Section 2 are applied to a periodic stochastic Lotka–Volterra competitive system with bounded delays. Section 4 investigates a periodic stochastic neural networks with infinite delay by using the theorems in Section 2.

2. Existence and uniqueness of the periodic Markov process

In this section, we will give some criterions on the existence and uniqueness of the periodic Markov process.

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space with the norm $|x|=\sqrt{\sum_{i=1}^n x_i^2}$ for $x\in\mathbb{R}^n$. Denote $\mathbb{R}_+=[0,\infty)$, $\mathbb{R}_-=(-\infty,0]$ and $\mathbb{R}^n_{++}=\{x\in\mathbb{R}^n : x_i>0, i=1,2,\cdots,n\}$. For $0<\tau<\infty$, denote by $C([-\tau,0],\mathbb{R}^n)$ the Banach space of continuous function $\phi(s):[-\tau,0]\to\mathbb{R}^n$ with the norm $\|\phi\|_C=\sup_{s\in[-\tau,0]}|\phi(s)|$ and $BC(\mathbb{R}_-,\mathbb{R}^n)$ the Banach space of bounded continuous function $\phi(s):\mathbb{R}_-\to\mathbb{R}^n$ with the norm $\|\phi\|_{BC}=\sup_{s\in\mathbb{R}_-}|\phi(s)|$. If $f(t)$ is a continuous bounded function on $\mathbb{R}_+$, we denote

$$\check{f}=\sup_{t\in\mathbb{R}_+} f(t) \quad \text{and} \quad \hat{f}=\inf_{t\in\mathbb{R}_+} f(t).$$

If $b_{ij}(t)$ and $c_i(t)$ are continuous bounded on $\mathbb{R}_+$ for $i,j=1,2,\cdots,n$, we denote $\tilde{b}_i=\max_{1\le j\le n}\check{b}_{ij}$, $\tilde{b}=\max_{1\le i,j\le n}\check{b}_{ij}$, $\bar{b}_i=\min_{1\le j\le n}\hat{b}_{ij}$, $\bar{b}=\min_{1\le i,j\le n}\hat{b}_{ij}$, $\tilde{c}=\max_{1\le i\le n}\check{c}_i$ and $\bar{c}=\min_{1\le i\le n}\hat{c}_i$.

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