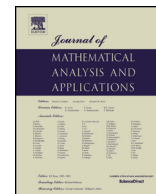




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# An algebraic approach to tempered ultradistributions <sup>☆</sup>

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## ABSTRACT

We construct the space of pseudo-quotients that is shown to be isomorphic to the spaces of Beurling tempered ultradistributions.

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## 1. Introduction

In our study, we were motivated by the results from [1] where an isomorphism between the space of tempered distributions and the space of pseudo-quotients including their convergence structures is presented. The main goal of our paper is to obtain such a result for the spaces of Beurling tempered ultradistributions. Our approach here is quite different than one in [1] which utilize the framework of pseudo-quotients. Rather it is based on an intrinsic analysis of a class of ultradifferential operators and corresponding structural theorems.

An open problem that still needs to be solved is to build a space of pseudo-quotients that is isomorphic to the space of Roumieu tempered ultradistributions.

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1.1. Notation

We employ the notation  $\mathbb{N}$ ,  $\mathbb{R}$  and  $\mathbb{C}$  for the sets of positive integers, real and complex numbers, respectively;  $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ . Given a multi-index  $n \in \mathbb{N}_0^d$ , for  $x \in \mathbb{R}^d$  we write as usual  $x^n = x_1^{n_1} \cdot \dots \cdot x_d^{n_d}$ ,  $D^n = D_x^n = D_1^{n_1} \cdot \dots \cdot D_d^{n_d}$ , where  $D_k = \frac{1}{i} \frac{\partial}{\partial x_k}$ ,  $k = 1, \dots, d$ , and for  $\alpha, \beta \in \mathbb{N}_0^d$ ,  $\alpha = (\alpha_1, \dots, \alpha_d), \beta = (\beta_1, \dots, \beta_d)$ ,  $\binom{\alpha}{\beta} = \prod_{k=1}^d \binom{\alpha_k}{\beta_k}$ ,  $\beta \leq \alpha$  means  $\beta_k \leq \alpha_k, k = 1, \dots, d$ ;  $|\alpha| = \alpha_1 + \dots + \alpha_d$ . We fix the constants in the Fourier transform as  $\mathcal{F}(f)(\xi) = \hat{f}(\xi) = \int_{\mathbb{R}^d} f(x) e^{i(x,\xi)} dx$ .

1.2. Ultradifferentiable functions of ultrapolynomial growth

Following the approach from [3] (cf. [2]), we introduce the test spaces for the spaces of tempered ultradistributions through the use of positive sequences  $(M_p)_{p \in \mathbb{N}_0}$  of real numbers which satisfy the following conditions:

- (M.1)  $M_p^2 \leq M_{p-1} M_{p+1}, p \in \mathbb{N}$ ;
- (M.2) There are constants  $A, H > 1$  such that  $M_p \leq AH^p \min_{0 \leq q \leq p} M_q M_{p-q}, p, q \in \mathbb{N}_0$ ;
- (M.3) There is a constant  $A$  such that  $\sum_{p=q+1}^\infty M_{p-1}/M_p < Aq M_q/M_{q+1}, q \in \mathbb{N}$ ;

We always assume  $M_0 = 1$ . The associated function for the sequence  $(M_p)_{p \in \mathbb{N}}$  is defined as

$$M(\lambda) = \sup_{p \in \mathbb{N}} \log \frac{\lambda^p}{M_p}, \lambda > 0 \text{ and } M(0) = 0.$$

We refer to [3] for the meaning of these three conditions and their translation into properties of  $M$ . In particular, the condition (M.2) is equivalent to

$$2M(\lambda) \leq M(H\lambda) + \ln A, \lambda > 0 \text{ (cf. [3, Proposition 3.6, p. 50])}. \tag{1.1}$$

Let  $m_p = M_p/M_{p-1}, p \in \mathbb{N}$ . We denote by  $m(\rho)$  the number of  $m_p \leq \rho$ . Then, we have

$$M(\lambda) = \int_0^\lambda \frac{m(\rho)}{\rho} d\rho, \lambda > 0 \text{ (cf. [3], (3.11), p. 50)}. \tag{1.2}$$

For the sake of simplicity, we consider  $M_p = p!^s$  with  $s > 1$ . In this case  $m_p = p^s$ . Hence, it follows from (1.2)

$$M(\lambda) \asymp s\lambda^{1/s}, \lambda > 0,$$

that is  $M(\lambda)/(s\lambda^{1/s}) \rightarrow 1$ , as  $\lambda \rightarrow \infty$ . Moreover, for every  $\varepsilon \in (0, 1)$  there exists  $\lambda_0$  such that  $(1 - \varepsilon)s\lambda^{1/s} \leq M(\lambda) \leq s\lambda^{1/s}, \lambda > \lambda_0$ .

Let  $h > 0$ . By  $\mathcal{S}_h^{(s)}(\mathbb{R}^d)$  we denote the Banach space of all smooth functions  $\varphi$  on  $\mathbb{R}^d$  such that the norm

$$\sigma_h(\varphi) = \sum_{n,k \in \mathbb{N}_0^d} \frac{h^{|n+k|}}{n!^s k!^s} \|x^n D^k \varphi\|_{L^2}$$

is finite. We define the space  $\mathcal{S}^{(s)}(\mathbb{R}^d)$  as a projective limit of the spaces  $\mathcal{S}_h^{(s)}(\mathbb{R}^d)$

$$\mathcal{S}^{(s)}(\mathbb{R}^d) = \varprojlim_{h \rightarrow \infty} \mathcal{S}_h^{(s)}(\mathbb{R}^d).$$

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