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Relationships between the oriented distance functional and a nonlinear separation functional ☆

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In this paper, we study two nonlinear scalarization functionals, namely, the oriented distance functional and a nonlinear separation functional due to Tammer. First, we prove the monotonicity of the oriented distance functional when the associated set is neither a cone nor a convex set. Then, we show several relationships between the two nonlinear scalarization functionals. By using these results, we also obtain an estimation of the sublevel set for the oriented distance functional.

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1. Introduction

Scalarization functionals are very important from the theoretical as well as computational points of view, and have been used in nonlinear analysis, vector optimization, economic equilibrium and related areas for a long period. However, it is not enough to use only linear scalarization functionals for studying nonconvex problems. So, several nonconvex separation functionals have been proposed. The most common scalarization functionals are the oriented distance functional and the nonlinear separation functional due to Tammer, and their many properties and applications have been shown in [1–20].

The nonlinear separation functional introduced by Tammer (Gerstewitz) [5] is widely used in vector optimization problems and related problems. This functional was also introduced and used in abstract convexity analysis under the name of topical function [14], or in mathematical finance under the name of coherent risk measure [9,12]. Its basic properties can be found in [6,7,12]. Recently, its Lipschitiz continuity, subdifferentiability, and sublevel set properties have also been investigated; see [1,3,8,15–17]. In addition, this functional is also applied to different research aspects of optimization theory, such as separation theorems for nonconvex sets [6], scalarization [6,8], optimality conditions [3,15], stability [1] in vector optimization.

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The oriented distance functional was proposed in nonlinear analysis by Hiriart-Urruty [10,11], and also used to study many optimization problems; see [2,4,13,18–20]. Its properties were studied by Hiriart-Urruty [10], Delfour and Zolésio [2], Zaffaroni [20], and Liu et al. [13]. In these literatures, authors obtained its subadditivity and monotonicity when the given set is a convex cone or convex set. Naturally, for neither a convex set nor a cone, these results need to be further studied for the oriented distance functional. Although the two nonlinear functionals are introduced in different fields, their properties are highly similar. Thus, people believe that they have certain connections. However, to our knowledge, there is no literature to discuss the relationships between the two nonlinear functionals.

Motivated by previous studies, we will investigate some basic properties of the oriented distance functional, and the relationships between the two nonlinear scalarization functionals. First, we discuss the monotonicity of the oriented distance functional in a nonconvex and noncone framework. Then, we show the relationships between the two nonlinear functionals under different assumptions. By using of these results, we establish an estimation of sublevel set for the oriented distance functional.

The rest of the paper is organized as follows. In Section 2, we recall some notations and definitions, and introduce the two nonlinear scalarization functionals. In Section 3, we investigate different monotonic properties of the oriented distance functional when the given set is neither a convex set nor a cone. In Section 4, we establish some relationships between the two nonlinear functionals, and obtain the estimation of sublevel set for the oriented distance functional.

2. Preliminaries

Let X be a topological vector space, and Y be a normed linear space with norm dual space Y^*. We denote by clA, $intA$ and bdA the closure, the interior and the boundary of a set $A \subset Y$, respectively. Let $D \subset Y$ be a proper (i.e., $D \neq Y$) closed convex cone, and $E \subset Y$ be a nonempty proper (i.e., $E \neq Y$) set with nonempty interior $intE$. The dual cone of D is denoted by $D^* := \{y^* \in Y^* : y^*(y) \geq 0, \forall y \in D\}$. We also assume that $S_Y = \{u \in Y : \|u\| = 1\}$ and $B(0,1) = \{u \in Y : \|u\| \leq 1\}$ are the unit sphere and closed unit ball in Y, respectively. $A^\infty := \{u \in Y : a + tu \in A \text{ for all } a \in A,\ t \in \mathbb{R}_+\}$ denotes the recession cone of $A \subset Y$. Obviously, $A + A^\infty = A$, A^∞ is a convex cone, and A^∞ is closed when A is closed.

For an arbitrary functional $\varphi : Y \to \mathbb{R} \cup \{\pm\infty\}$, we denote

$$\begin{aligned} \operatorname{dom} \varphi &:= \{y \in Y : \varphi(y) < +\infty\}, \\ \partial\varphi(\bar{y}) &:= \{y^* \in Y^* : \varphi(y) - \varphi(\bar{y}) \geq y^*(y - \bar{y}),\ \forall y \in Y\}. \end{aligned}$$

Let $q \in Y \backslash \{0\}$. We define the functional $\varphi_E^q : Y \to \mathbb{R} \cup \{\pm\infty\}$ as follows:

$$\varphi_E^q(y) := \begin{cases} +\infty & \text{if } \ y \notin \mathbb{R}q + E, \\ \inf\{t \in \mathbb{R} : y + tq \in E\} & \text{otherwise.} \end{cases}$$

Several basic properties of φ_E^q were collected in books [7,12]. Other properties of φ_E^q were studied further in recent years, such as Lipschitiz continuity [1,15], subdifferentiability [3], sublevel set properties [16,17]. Especially in literatures [8,16], the authors discussed the properties of φ_E^q when E is q-vectorially closed.

Next, we introduce another important nonconvex separation functional. The so-called oriented distance functional (Hiriart-Urruty functional) $\Delta_E : Y \to \mathbb{R} \cup \{\pm\infty\}$ is defined as follows:

$$\Delta_E(v) = d(v, E) - d(v, Y \backslash E)$$

where $d(v, E) = \inf_{z \in E} \|v - z\|$, $d(v, \emptyset) = +\infty$.

The oriented distance functional Δ_E has many good properties, and its detailed discussion can be seen in [2,10,11,13,20].

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