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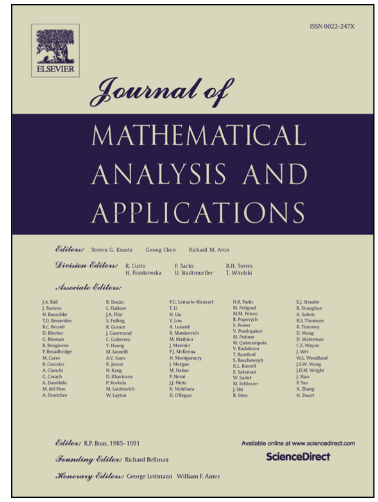

A dissipative hyperbolic affine curve flow

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A dissipative hyperbolic affine curve flow

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Abstract: A new hyperbolic affine geometric flow with dissipative term is proposed. The equations satisfied by support functions and the graph of the curve under this dissipative flow give rise to fully nonlinear hyperbolic equations, we obtain the existence for local solutions of this flow by reducing the flow to a first-order system. The equation for both perimeter and area of closed curves under the flow are also obtained. Based on this, we show that for a closed curve, the solution of this flow converges to a point in finite time. Furthermore, by a method of LeFloch-Smocyk for studying the hyperbolic mean curvature flow, global existence of the solution is established.

MSC:58J45,58J47

Keywords: Hyperbolic affine curve flow with dissipative term; Nonlinear hyperbolic equations; Blow up; Global existence

1. Introduction

In this paper we study the hyperbolic version of affine geometric flow with dissipative term. More precisely, we consider Cauchy problem of the hyperbolic affine planar curve flow

$$\begin{cases} \dfrac{\partial^2\gamma}{\partial t^2}+\beta\dfrac{\partial\gamma}{\partial t}=\mathcal{N} \\ \gamma(p,0)=\gamma_0(p), \\ \dfrac{\partial\gamma}{\partial t}(p,0)=\gamma_1(p), \end{cases} \tag{1.1}$$

where $\beta\geq 0$ is a constant, $\mathcal{N}$ is the affine normal, which is related to the Euclidean unit normal $\mathbf{n}$ and tangent vector $\mathbf{t}$ via $\mathcal{N}=k^{1/3}\mathbf{n}-\frac{1}{3}k^{-5/3}k_s\mathbf{t}$, k is Euclidean curvature, and s is the Euclidean arc-length.

The parabolic theory for the evolving of plane curves has been extremely successful in providing geometers with great insight (see Gage and Hamilton [8], Grayson [9] and Zhu [25]). It can be applied to many physical problems such as crystal growth, computer vision and image processing [5]. In particular, there are many interesting works (see [7], [8], [9] and references therein) which consider the Euclidean curve-shortening problem

$$\begin{cases} \dfrac{\partial\gamma}{\partial t}=k\mathbf{n}=\dfrac{\partial^2\gamma}{\partial s^2} \\ \gamma(p,0)=\gamma_0(p), \end{cases} \tag{1.2}$$

where $\gamma(p,t)\in\mathbb{R}^2$ is a family of closed curves and s is the Euclidean arc-length parameter of the curve. Since the affine curve-shortening flow has important applications in image

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