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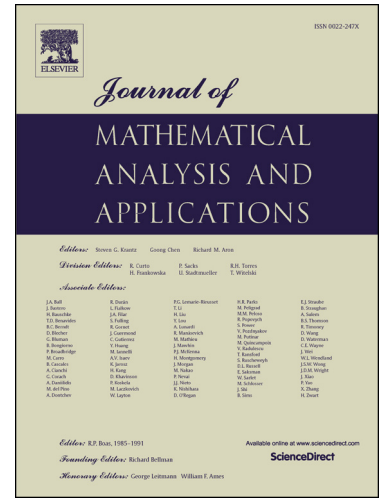
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HEAT KERNELS FOR TIME-DEPENDENT NON-SYMMETRIC STABLE-LIKE OPERATORS

Zhen-Qing Chen and Xicheng Zhang

ABSTRACT. When studying non-symmetric nonlocal operators on \mathbb{R}^d :

$$\mathcal{L}f(x) = \int_{\mathbb{R}^d} (f(x+z) - f(x) - \nabla f(x) \cdot z 1_{\{|z| \leq 1\}}) \frac{\kappa(x, z)}{|z|^{d+\alpha}} dz,$$

where $0 < \alpha < 2$, $d \geq 1$, and $\kappa(x, z)$ is a function on $\mathbb{R}^d \times \mathbb{R}^d$ that is bounded between two positive constants, it is customary to assume that $\kappa(x, z)$ is symmetric in z . In this paper, we study heat kernel of \mathcal{L} and derive its two-sided sharp bounds without the symmetric assumption $\kappa(x, z) = \kappa(x, -z)$. In fact, we allow the kernel κ to be time-dependent and $x \rightarrow \kappa(t, x, z)$ to be only locally β -Hölder continuous with Hölder constant possibly growing at a polynomial rate in $|z|$. We also derive gradient estimate when $\beta \in (0 \vee (1 - \alpha), 1)$ as well as fractional derivative estimate of order $\theta \in (0, (\alpha + \beta) \wedge 2)$ for the heat kernel. Moreover, when $\alpha \in (1, 2)$, drift perturbation of the time-dependent non-local operator \mathcal{L}_t with drift in Kato's class is also studied in this paper. As an application, when $\kappa(x, z) = \kappa(z)$ does not depend on x , we show the boundedness of nonlocal Riesz's transformation: for any $p > 2d/(d + 2\alpha)$,

$$\|\mathcal{L}^{1/2} f\|_p \asymp \|\Gamma(f)^{1/2}\|_p,$$

where $\Gamma(f) := \frac{1}{2}\mathcal{L}(f^2) - f\mathcal{L}f$ is the carré du champ operator associated with \mathcal{L} , and $\mathcal{L}^{1/2}$ is the square root operator of \mathcal{L} defined by using Bochner's subordination. Here \asymp means that both sides are comparable up to a constant multiple.

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Keywords and Phrases: Heat kernel estimates, non-symmetric nonlocal operator, Levi's method, Riesz's transform

1. INTRODUCTION

Let $\alpha \in (0, 2)$ and $d \geq 1$. We consider the following time-dependent nonlocal and non-symmetric operator on \mathbb{R}^d :

$$\mathcal{L}_t^\kappa f(x) := \int_{\mathbb{R}^d} (f(x+z) - f(x) - z^{(\alpha)} \cdot \nabla f(x)) \frac{\kappa(t, x, z)}{|z|^{d+\alpha}} dz, \quad (1.1)$$

where

$$z^{(\alpha)} := (1_{\alpha \in (1, 2)} + 1_{|z| \leq 1} 1_{\alpha=1}) z, \quad (1.2)$$

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