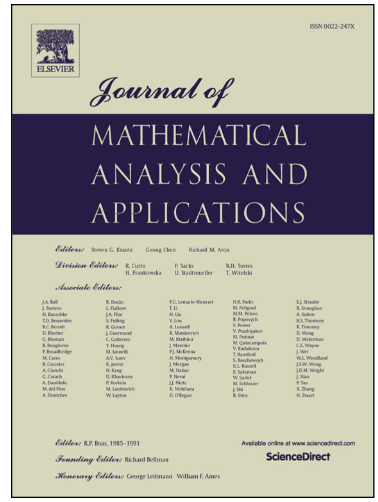
Accepted Manuscript

Heat kernels for time-dependent non-symmetric stable-like operators

Zhen-Qing Chen, Xicheng Zhang



PII: S0022-247X(18)30266-X
DOI: https://doi.org/10.1016/j.jmaa.2018.03.054
Reference: YJMAA 22130

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 15 September 2017

Please cite this article in press as: Z.-Q. Chen, X. Zhang, Heat kernels for time-dependent non-symmetric stable-like operators, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2018.03.054

HEAT KERNELS FOR TIME-DEPENDENT NON-SYMMETRIC STABLE-LIKE OPERATORS

Zhen-Qing Chen and **Xicheng Zhang**

Abstract. When studying non-symmetric nonlocal operators on $\mathbb{R}^d$:

$$\mathscr{L} f(x) = \int_{\mathbb{R}^d} \left(f(x+z) - f(x) - \nabla f(x) \cdot z 1_{\{|z|\leqslant 1\}} \right) \frac{\kappa(x,z)}{|z|^{d+\alpha}} \mathrm{d}z,$$

where $0 < \alpha < 2$, $d \geqslant 1$, and $\kappa(x,z)$ is a function on $\mathbb{R}^d \times \mathbb{R}^d$ that is bounded between two positive constants, it is customary to assume that $\kappa(x,z)$ is symmetric in z. In this paper, we study heat kernel of $\mathscr{L}$ and derive its two-sided sharp bounds without the symmetric assumption $\kappa(x,z) = \kappa(x,-z)$. In fact, we allow the kernel κ to be time-dependent and $x \to \kappa(t,x,z)$ to be only locally β-Hölder continuous with Hölder constant possibly growing at a polynomial rate in $|z|$. We also derive gradient estimate when $\beta \in (0 \vee (1-\alpha), 1)$ as well as fractional derivative estimate of order $\theta \in (0, (\alpha+\beta) \wedge 2)$ for the heat kernel. Moreover, when $\alpha \in (1,2)$, drift perturbation of the time-dependent non-local operator $\mathscr{L}_t$ with drift in Kato's class is also studied in this paper. As an application, when $\kappa(x,z) = \kappa(z)$ does not depend on x, we show the boundedness of nonlocal Riesz's transorfmation: for any $p > 2d/(d+2\alpha)$,

$$\|\mathscr{L}^{1/2} f\|_p \asymp \|\Gamma(f)^{1/2}\|_p,$$

where $\Gamma(f) := \frac{1}{2}\mathscr{L}(f^2) - f\mathscr{L} f$ is the carré du champ operator associated with $\mathscr{L}$, and $\mathscr{L}^{1/2}$ is the square root operator of $\mathscr{L}$ defined by using Bochner's subordination. Here $\asymp$ means that both sides are comparable up to a constant multiple.

AMS 2010 Mathematics Subject Classification: Primary 35K05, 60J35, 47G20; Secondary 47D07

Keywords and Phrases: Heat kernel estimates, non-symmetric nonlocal operator, Levi's method, Riesz's transform

1. Introduction

Let $\alpha \in (0,2)$ and $d \geqslant 1$. We consider the following time-dependent nonlocal and non-symmetric operator on $\mathbb{R}^d$:

$$\mathscr{L}^\kappa_t f(x) := \int_{\mathbb{R}^d} \Big(f(x+z) - f(x) - z^{(\alpha)} \cdot \nabla f(x) \Big) \frac{\kappa(t,x,z)}{|z|^{d+\alpha}} \mathrm{d}z, \tag{1.1}$$

where

$$z^{(\alpha)} := \big(1_{\alpha\in(1,2)} + 1_{|z|\leqslant 1} 1_{\alpha=1} \big) z, \tag{1.2}$$

This work is partially supported by Simons Foundation grant 520542 and by an NNSFC grant of China (No. 11731009).