

**AN OPTIMAL INEQUALITY FOR LAGRANGIAN
SUBMANIFOLDS IN COMPLEX SPACE FORMS INVOLVING
CASORATI CURVATURE**

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ABSTRACT. In this paper, we establish an optimal inequality involving normalized δ -Casorati curvature $\delta_C(n-1)$ of Lagrangian submanifolds in n -dimensional complex space forms. We derive a very singular and unexpected result: the lower bounds of the normalized δ -Casorati curvatures $\delta_C(n-1)$ and $\widehat{\delta}_C(n-1)$ in terms of dimension, the holomorphic sectional curvature, the normalized scalar curvature and the squared mean curvature of the submanifold, are different, in contrast to all previous results obtained for several classes of submanifolds in many ambient spaces. We also investigate the equality case of the inequality and prove that a Casorati $\delta_C(n-1)$ -ideal Lagrangian submanifold of a complex space form without totally geodesic points is an H -umbilical Lagrangian submanifold of ratio 4. Some examples are discussed in the last part of the paper, showing that the constants in the inequality obtained in this work are the best possible.

Keywords: Casorati curvature, Lagrangian submanifold, complex space form, ideal submanifold, H -umbilical submanifold.

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1. INTRODUCTION

In the early nineties, in order to provide answers to an open question raised by Chern concerning the existence of minimal immersions into an Euclidean space of arbitrary dimension, B.-Y. Chen introduced new types of Riemannian invariants, called δ -invariants or Chen invariants, and established sharp inequalities, known as Chen's inequalities, involving these new intrinsic invariants and the main extrinsic invariants for arbitrary Riemannian submanifolds. Due to the fact that one of the most fundamental problems in the theory of submanifolds is to obtain simple relationships between intrinsic and extrinsic quantities, the study of δ -invariants has immediately attracted a lot of attention and Chen-like inequalities were obtained by many authors for several classes of submanifolds in different (pseudo-)Riemannian spaces (see, e.g., [5] and the references therein). We recall that the equality case of the Chen's inequalities leads to the notions of *ideal submanifold* and *the best ways of living*, concepts which are well motivated because ideal submanifolds are those submanifolds which receive the least possible tension at each point from the ambient space [6, 10]. Later, some other invariants of similar nature were introduced by several geometers and also called δ -invariants. Among them, we have the δ -Casorati curvatures which are defined using the Casorati curvature of hyperplanes in the tangent space at a point. It is known that the Casorati curvature of a submanifold

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