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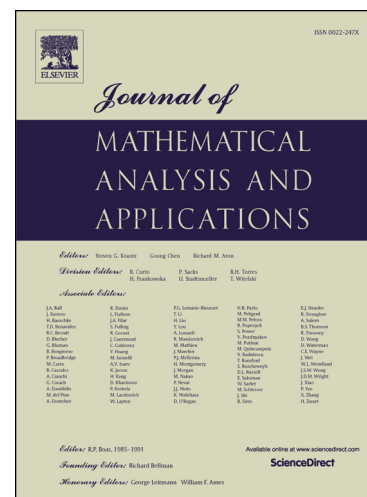
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**SOME OVERDETERMINED PROBLEMS RELATED TO THE  
ANISOTROPIC CAPACITY**

CHIARA BIANCHINI, GIULIO CIRAIOLO, AND PAOLO SALANI

ABSTRACT. We characterize the Wulff shape of an anisotropic norm in terms of solutions to overdetermined problems for the Finsler  $p$ -capacity of a convex set  $\Omega \subset \mathbb{R}^N$ , with  $1 < p < N$ . In particular we show that if the Finsler  $p$ -capacitary potential  $u$  associated to  $\Omega$  has two homothetic level sets then  $\Omega$  is Wulff shape. Moreover, we show that the concavity exponent of  $u$  is  $\mathbf{q} = -(p-1)/(N-p)$  if and only if  $\Omega$  is Wulff shape.

**AMS subject classifications.** 35N25, 35B06, 35R25.

**Key words.** Wulff shape. Overdetermined problems. Capacity. Concavity exponent.

1. INTRODUCTION

The aim of this paper is to study some unconventional overdetermined problems for the Finsler  $p$ -capacity of a bounded convex set  $\Omega$  associated to a norm  $H$  of  $\mathbb{R}^N$ ,  $N \geq 3$ .

Given a bounded convex domain  $\Omega \subset \mathbb{R}^N$ , the  $p$ -capacity of  $\Omega$  is defined by

$$\text{Cap}_p(\Omega) = \inf \left\{ \frac{1}{p} \int_{\mathbb{R}^N} |D\varphi|^p dx, \varphi \in C_0^\infty(\mathbb{R}^N), \varphi(x) \geq 1 \text{ for } x \in \Omega \right\}$$

with  $1 < p < N$ . When the Euclidean norm  $|\cdot|$  is replaced by a more general norm  $H(\cdot)$ , one can consider the so called *Finsler  $p$ -capacity*  $\text{Cap}_{H,p}(\Omega)$ , which is defined by

$$\text{Cap}_{H,p}(\Omega) = \inf \left\{ \frac{1}{p} \int_{\mathbb{R}^N} H^p(D\varphi) dx, \varphi \in C_0^\infty(\mathbb{R}^N), \varphi(x) \geq 1 \text{ for } x \in \Omega \right\}, \quad (1.1)$$

for  $1 < p < N$ . Under suitable assumptions on the norm  $H$  and on the set  $\Omega$ , the above infimum is attained and

$$\text{Cap}_{H,p}(\Omega) = \frac{1}{p} \int_{\mathbb{R}^N} H^p(Du_\Omega) dx,$$

where  $u_\Omega$  is the solution of the Finsler  $p$ -capacity problem

$$\begin{cases} \Delta_p^H u = 0 & \text{in } \mathbb{R}^N \setminus \bar{\Omega}, \\ u = 1 & \text{on } \partial\Omega, \\ u \rightarrow 0 & \text{as } H(x) \rightarrow +\infty. \end{cases} \quad (1.2)$$

Here  $\Delta_p^H$  denotes the Finsler  $p$ -Laplace operator, i.e.  $\Delta_p^H u = \text{div}(H^{p-1}(Du)\nabla H(Du))$ . The function  $u_\Omega$  is named (*Finsler*)  $p$ -capacitary potential of  $\Omega$ .

When  $\Omega$  is Wulff shape, i.e. it is a sublevel set of the dual norm  $H_0$

$$\Omega = B_{H_0}(r) = \{x \in \mathbb{R}^N : H_0(x - \bar{x}) < r\}$$

(see Section 2 for definitions), the solution to (1.2) can be explicitly computed and it is given by

$$v_r(x) = \left( \frac{H_0(x - \bar{x})}{r} \right)^{\frac{1}{\mathbf{q}}}, \quad (1.3)$$

with

$$\mathbf{q} = -\frac{p-1}{N-p}. \quad (1.4)$$

It is straightforward to verify that the potential  $v_r$  in (1.3) enjoys the following properties:

- (i) the function  $v_r^{\mathbf{q}}$  is convex, i.e.  $v_r$  is  $\mathbf{q}$ -concave;
- (ii) the superlevel sets of  $v_r$  are homothetic sets and they are Wulff shapes;
- (iii)  $H(Dv_r)$  is constant on the level sets of  $v_r$ .

The aim of this paper is to show that each of the properties (i)-(iii) characterizes the Wulff shape under some regularity assumptions on the norm  $H$  and on  $\Omega$ . In particular, we assume that  $H \in \mathcal{J}_p$  where

$$\mathcal{J}_p = \{H \in C_+^2(\mathbb{R}^N \setminus \{0\}), H^p \in C^{2,1}(\mathbb{R}^N \setminus \{0\})\}. \quad (1.5)$$

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