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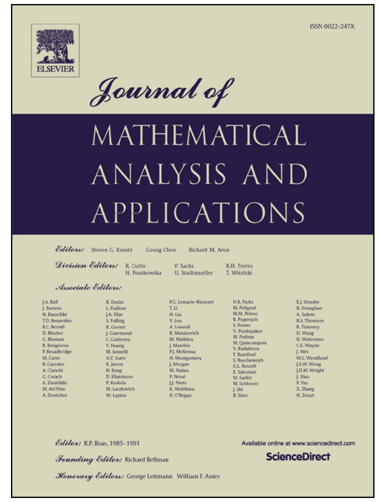

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ACCEPTED MANUSCRIPT

MORE ACCURATE OPERATOR MEANS INEQUALITIES

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Abstract. Our main target in this paper is to present new sharp bounds for inequalities that result when weighted operator means are filtered through positive linear maps and operator monotone functions.

As an application, we prove a refined reverse of the celebrated Golden-Thompson inequality.

Furthermore, we show how these inequalities can be squared.

In order to prove our results, we will need to show a new operator arithmetic-geometric mean inequality. Manipulating this operator inequality will enable us to present the new inequalities for positive linear maps and operator monotone functions.

1. Introduction and Motivation

Let $(\mathcal{H}, \langle\cdot,\cdot\rangle)$ be a complex Hilbert space and let $\mathcal{B}(\mathcal{H})$ denote the algebra of all bounded linear operators acting on $\mathcal{H}$. An operator T is said to be *positive* (denoted by $T \geq 0$) if $\langle Tx, x\rangle \geq 0$ for all $x \in \mathcal{H}$, and T is said to be *strictly positive* (denoted by $T > 0$) if T is positive and invertible. A real-valued continuous function f defined on $[0,\infty)$ is called *operator monotone* (resp. *operator monotone decreasing*) if $f(A) \geq f(B)$ (resp. $f(B) \geq f(A)$) for $A \geq B \geq 0$. In this context, we say that $A \geq B$ when $A - B \geq 0$.

A linear map $\Phi : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ is said to be positive if $\Phi(A) \geq 0$ when $A \geq 0$. It is said to be normalized (or unital) if it maps the identity I to itself.

As a matter of convenience, we use the following notations to define the v-weighted harmonic mean, geometric mean, and arithmetic mean for scalars and operators:

$$a!b = \left((1-v)\frac{1}{a} + v\frac{1}{b}\right)^{-1}, \qquad a\sharp_v b = a^{1-v}b^v, \qquad a\nabla_v b = (1-v)a + vb,$$

$$A!_vB = \left((1-v)A^{-1} + vB^{-1}\right)^{-1}, \qquad A\sharp_v B = A^{\frac{1}{2}}\left(A^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)^v A^{\frac{1}{2}}, \qquad A\nabla_v B = (1-v)A + vB,$$

where $a, b > 0$ and $A, B \in \mathcal{B}(\mathcal{H})$ are strictly positive operators and $v \in [0,1]$.

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