

# Accepted Manuscript

Lower bounds for the Lipschitz constants of some classical fixed point free maps

J. Ferrer, E. Llorens-Fuster

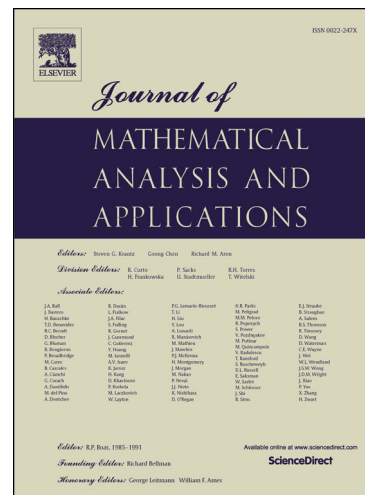
PII: S0022-247X(18)30382-2  
DOI: <https://doi.org/10.1016/j.jmaa.2018.05.001>  
Reference: YJMAA 22229

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 11 May 2017

Please cite this article in press as: J. Ferrer, E. Llorens-Fuster, Lower bounds for the Lipschitz constants of some classical fixed point free maps, *J. Math. Anal. Appl.* (2018), <https://doi.org/10.1016/j.jmaa.2018.05.001>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# Lower bounds for the Lipschitz constants of some classical fixed point free maps

J. Ferrer\* and E. Llorens-Fuster †

May 3, 2018

*Dedicated to the memory of our colleague and friend Bernardo Cascales.*

## Abstract

We find lower bounds for the set of Lipschitz constants of a given Lipschitzian map, defined on the closed unit ball of a Hilbert space, with respect to any renorming. We introduce a class of maps, defined in the closed unit ball of  $\ell_2$ , which contains the classical fixed point free maps due to Goebel-Kirk-Thelle, Baillon, and P. K. Lin. We show that for any map of this class its uniform Lipschitz constant with respect to any renorming of  $\ell_2$  is never strictly less than  $\frac{\pi}{2}$ .

*Keywords:* Hilbert space, Fixed point, Fixed point free mapping, Lipschitzian map, Lipschitz constant

## 1 Introduction

An old open problem in metric Fixed Point Theory is to know whether there exists a reflexive Banach space  $X$  such that for some closed convex bounded subset  $K$  of  $X$  one can find a self-map  $T : K \rightarrow K$  such that it has no fixed points although it can be nonexpansive, i.e., 1-Lipschitzian, with respect to some equivalent renorming of  $X$ .

Since 1965 many authors have found partial negative answers to this problem, mainly by means of giving geometrical properties which imply that every selfmapping of  $K$  which is nonexpansive with respect to a renorming enjoying these properties must have a fixed point.

A different strategy to solve this problem is to start with examples of fixed point free self-maps of, for instance, the closed unit ball of  $\ell_2$ , and then try to find a suitable renorming of  $\ell_2$  with respect to which the map under consideration could become nonexpansive. (See [3, 4, 11, 10] and the references therein).

---

\*The first author has been supported by MINECO and FEDER Project MTM2014-57838-C2-2-P.

†The second author has been supported by grant MTM2015-65242-C2-2P.

Download English Version:

<https://daneshyari.com/en/article/8899500>

Download Persian Version:

<https://daneshyari.com/article/8899500>

[Daneshyari.com](https://daneshyari.com)