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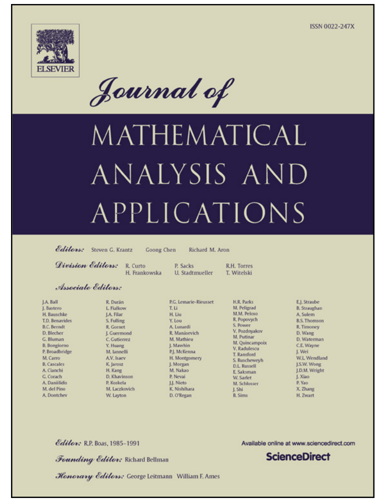

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Lower bounds for the Lipschitz constants of some classical fixed point free maps

J. Ferrer* and E. Llorens-Fuster †

May 3, 2018

Dedicated to the memory of our colleague and friend Bernardo Cascales.

Abstract

We find lower bounds for the set of Lipschitz constants of a given Lipschitzian map, defined on the closed unit ball of a Hilbert space, with respect to any renorming. We introduce a class of maps, defined in the closed unit ball of ℓ_2, which contains the classical fixed point free maps due to Goebel-Kirk-Thelle, Baillon, and P. K. Lin. We show that for any map of this class its uniform Lipschitz constant with respect to any renorming of ℓ_2 is never strictly less than $\frac{\pi}{2}$.

Keywords: Hilbert space, Fixed point, Fixed point free mapping, Lipschitzian map, Lipschitz constant

1 Introduction

An old open problem in metric Fixed Point Theory is to know wether there exists a reflexive Banach space X such that for some closed convex bounded subset K of X one can find a self-map $T : K \to K$ such that it has no fixed points although it can be nonexpansive, i.e., 1-Lipschitzian, with respect to some equivalent renorming of X.

Since 1965 many authors have found partial negative answers to this problem, mainly by means of giving geometrical properties which imply that every selfmappig of K which is nonexpansive with respect to a renorming enjoying these properties must have a fixed point.

A different strategy to solve this problem is to start with examples of fixed point free self-maps of, for instance, the closed unit ball of ℓ_2, and then try to find a suitable renorming of ℓ_2 with respect to which the map under consideration could become nonexpansive. (See [3, 4, 11, 10] and the references therein).

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