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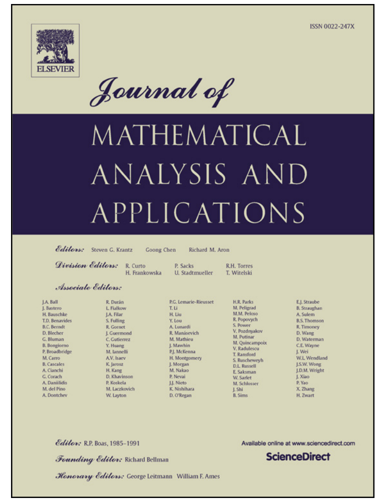

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Completely linear degeneracy for quasilinear hyperbolic systems

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Abstract

In this paper, we introduce a new concept of completely linear degeneracy for quasilinear hyperbolic systems in several space variables, and then get an interesting property for multidimensional hyperbolic conservation laws satisfying our new definition. For applications, we give some examples arising from mathematics and physics at last.

1 Introduction

Quasilinear hyperbolic systems in several space variables can be described as follows

$$\frac{\partial u}{\partial t} + \sum_{j=1}^{m} A_j(u) \frac{\partial u}{\partial x_j} = 0, \tag{1.1}$$

where $u = (u_1, \cdots, u_n)^T$ is the unknown vector function of $(t, x_1, \cdots, x_m)$ and $A_j(u)\,(j = 1, \cdots, m)$ is an $n \times n$ matrix with smooth elements $a_{jkl}(u)\,(k, l = 1, \cdots, n)$. The concepts of linear degeneracy and genuine nonlinearity have been made in the following way (see [24]). This is a straightforward generalization of the case in one space dimension (see [20]). The i-th characteristic field of system (1.1) is linearly degenerate, if

$$\nabla \lambda_i(u, \xi) \cdot r_i(u, \xi) \equiv 0, \quad \forall\, u \in \Omega, \ \ \forall\, \xi \in \mathbb{S}^{m-1}; \tag{1.2}$$

while, it is genuinely nonlinear, if

$$\nabla \lambda_i(u, \xi) \cdot r_i(u, \xi) \neq 0, \quad \forall\, u \in \Omega, \ \ \forall\, \xi \in \mathbb{S}^{m-1}, \tag{1.3}$$

where $\xi = (\xi_1, \cdots, \xi_m)^T \in \mathbb{S}^{m-1}$, $\lambda_1(u, \xi), \cdots, \lambda_n(u, \xi)$ are n real eigenvalues of $A(u, \xi) = \sum_{j=1}^{m} A_j(u)\xi_j$ and $\{r_i(u, \xi)\}_{i=1}^{n}$ is a complete set of right eigenvectors of $A(u, \xi)$. Here and throughout this paper, we always assume that $A(u, \xi)$ has n real eigenvalues. However, as is pointed out in [21], this generalization would exclude a single equation and a system of two

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