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Recovering functions from the spherical mean transform with limited radii data by expansion into spherical harmonics

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ABSTRACT

The aim of the article is to generalize the method presented in [3, Theorem 1] by G. Ambartsoumian, R. Gouia-Zarrad and M. Lewis for recovering functions from their spherical mean transform with limited radii data from the two dimensional case to the general n dimensional case. The idea behind the method is to expand each function in question into spherical harmonics and then obtain, for each term in the expansion, an integral equation of Volterra's type that can be solved iteratively. We show also how this method can be modified for the spherical case of recovering functions from the spherical transform with limited radii data. Lastly, we solve the analogous problem for the case of the Funk transform by again using expansion into spherical harmonics and then obtain an Abel type integral equation which can be inverted by a method introduced in [14].

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1. Introduction and motivation

The aim of this paper is to provide new inversion methods and algorithms for recovering a function f from its spherical mean transform which integrates f on a family of spheres with a prescribed set of centers and radii.

The spherical mean transform has been found out to be a relatively accurate modeling of the heat propagation which occurs in thermoacoustic tomography when a short duration electromagnetic pulse is sent through an object Ω in space (see [4,6,7,9,10,12,16,18]). The heat propagation is measured by transducers located on a surface Γ which surrounds the scattered object Ω such that at any time $t \geq 0$ each transducer receives the heat data from points which all have an equal distance from its location. This information can be translated into an integration of a modeling function f on a family of spheres with centers on the surface Γ which clearly translates to the spherical mean transform of f.

Observe that at each location x of a transducer on the surface Γ, the data received from the heat propagation can be measured at any time $t \geq 0$. Thus, integration of the modeling function f can be done on each sphere with a center on the surface Γ and with an arbitrary radius. Inversion formulas for recovering

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a function f from its spherical mean transform where the family of the spheres of integration is described as above (centers on a closed surface, arbitrary radius) have been obtained, for example, in [5,8–10,13,19] where Γ is a sphere, in [11,15] where Γ is an ellipsoid and in [11,19] where Γ is a cylinder.

However, in case where some constraints are imposed on the experimental modeling, such as a time limit of the experiment or a limited sensitivity of each transducer, one has to assume as a consequence that integration of f can be done on spheres whose radii belong to a prescribed set. This raises the problem of recovering a function f from its spherical mean transform where the spheres of integration have centers on a closed surface Γ and with radii which belong to a finite interval. This problem was first addressed in [3] by G. Ambartsoumian, R. Gouia-Zarrad and M. Lewis.

In [3] an inversion method was obtained, in the two dimensional case, for recovering a function f from its spherical mean transform in case where Γ is a circle with the center at the origin and radius R and such that the radius of each circle of integration is assumed to belong to the interval $[0, R - \epsilon]$ where $0 < \epsilon < R$.

The method for the inversion as was introduced in [3] consists mainly of two steps. In the first step, the function f in question is expanded into its Fourier series in $\mathbb{R}^2$. Then, one can use the rotation invariance of our problem in order to reduce it to each term in the expansion. In the second step, one can convert the inversion problem, for each term in the expansion, into the problem of solving a Volterra equation of the second kind and use known methods (such as in [17]) in order to solve this equation which results in extracting the corresponding term of the expansion.

In the first result of this paper, Theorem 3.1, we generalize the results and methods obtained in [3] for the general n dimensional case. For this, we expand each function in question into spherical harmonics (for $n = 2$ this is just the Fourier series expansion used in [3]) and as in [3] we can reduce our problem to each term in the expansion. Then we show how the reduced problem, for each term in the expansion, can be converted into the problem of solving a Volterra equation of the second kind by modifying the method used in [3] for the cases where n is even or odd.

In the second main result of this paper, Theorem 3.3, we modify the method used in the proof of Theorem 3.1, which recovers functions defined on the Euclidean space $\mathbb{R}^n$, in order to obtain similar results for the spherical case. That is, for a function f, defined on the unit sphere $\mathbb{S}^{n-1}$ in $\mathbb{R}^n$, we define the spherical transform which is an analogous integral transform to the spherical mean transform for functions defined on $\mathbb{S}^{n-1}$. Then, we show how to obtain an inversion method to the limited radii problem for this case.

In the third and last result of this paper, Theorem 3.4, we show how to derive an inversion method for the Funk transform, which integrates functions defined on $\mathbb{S}^{n-1}$ on great subspheres, with limited data on the family of the subspheres of integration. For this case the great subspheres of integration are assumed to be disjointed from some fixed upper and lower spherical caps and our aim is to recover a function f, defined on $\mathbb{S}^{n-1}$, from this data on the complement of these caps in $\mathbb{S}^{n-1}$. Since this limited data problem is somewhat conceptually different from the limited data problem introduced in Theorem 3.1 and 3.3, the method of inversion is also somewhat different. As in the previous cases we expand each function f in question into spherical harmonics and use the rotation invariance, with respect to the unit vector e_n, of our problem in order to reduce it to each term in the expansion. However, contrary to the previous cases, where our problem was reduced to solve a Volterra integral equation of the second kind, in this case our problem is reduced to the problem of solving an Abel type integral equation which can be inverted by a method introduced in [14].

The author of this paper was informed very recently that the result of Theorem 3.1 was also proved by G. Ambartsoumian, R. Gouia-Zarrad, V. Krishnan and S. Roy in [2, Theorem 2.2]. Similarly to this paper, the authors expand each function f in question into spherical harmonics and reduce the problem of recovering f into the problem of recovering each term, in the expansion of f into spherical harmonics, from a Volterra integral equation of the second kind.

As a first step for the reduction to the Volterra equation the authors first express the infinitesimal volume measure of each sphere, on which f is integrated, via the infinitesimal volume measure of the sphere S of

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