



Γ -limits of singular perturbation problems involving energies with non-local terms

Arkady Poliakovsky

Department of Mathematics, Ben Gurion University of the Negev, P.O.B. 653, Be'er Sheva 84105, Israel

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ABSTRACT

We introduce a new method to reformulate certain classes of problems involving non-local terms as local problems. This allows us to apply the techniques developed in [24,25] to prove upper and lower bounds for problems arising in Micromagnetics and in the variational study of the Method of Vanishing Viscosity for systems of conservation laws.

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1. Introduction

1.1. Problems with a non-local term

Let $G \in C^1(\mathbb{R}^{m \times N^n} \times \mathbb{R}^{m \times N^{(n-1)}} \times \dots \times \mathbb{R}^{m \times N} \times \mathbb{R}^m, \mathbb{R})$ and $W \in C^1(\mathbb{R}^m, \mathbb{R})$ be nonnegative functions such that $G(0, 0, \dots, 0, b) = 0$ and let $\Psi \in C^1(\mathbb{R}^m, \mathbb{R}^{l \times N})$. Consider the energy functional with a nonlocal term, defined for every $\varepsilon > 0$ by

$$I_\varepsilon(\phi) = \int_{\Omega} \frac{1}{\varepsilon} G(\varepsilon^n \nabla \phi^n, \dots, \varepsilon \nabla \phi, \phi) dx + \int_{\Omega} \frac{1}{\varepsilon} W(\phi) dx + \frac{1}{\varepsilon} \int_{\mathbb{R}^N} |\nabla \bar{H}_{\Psi(\phi)}|^2 dx \quad \text{for } \phi : \Omega \rightarrow \mathcal{M} \subset \mathbb{R}^m. \quad (1.1)$$

In (1.1), $u : \Omega \rightarrow \mathbb{R}^{l \times N}$ and $\bar{H}_u : \mathbb{R}^N \rightarrow \mathbb{R}^l$ is defined by

$$\begin{cases} \Delta \bar{H}_u = \operatorname{div} \{ \chi_{\Omega} u \} & \text{in the sense of distributions in } \mathbb{R}^N, \\ \nabla \bar{H}_u \in L^2(\mathbb{R}^N, \mathbb{R}^{l \times N}), \end{cases} \quad (1.2)$$

E-mail address: poliakov@math.bgu.ac.il.

where χ_Ω is the characteristic function of Ω . Functionals of the type (1.1) are relevant in Micromagnetics (see [1], [12], [26], [27]). The full three-dimensional model of ferromagnetic materials deals with an energy functional, which, up to a rescaling, has the form

$$E_\varepsilon(m) := \varepsilon \int_{\Omega} |\nabla m|^2 dx + \frac{1}{\delta_\varepsilon} \int_{\Omega} W(m) dx + \frac{1}{\varepsilon} \int_{\mathbb{R}^3} |\nabla \bar{H}_m|^2 dx, \quad (1.3)$$

where $\Omega \subset \mathbb{R}^3$ is a bounded domain, $m : \Omega \rightarrow S^2$ stands for the magnetization, $\delta_\varepsilon > 0$ is a material parameter and $\bar{H}_m : \mathbb{R}^3 \rightarrow \mathbb{R}$ is defined, as above, by

$$\begin{cases} \Delta \bar{H}_m = \operatorname{div} \{ \chi_\Omega m \} & \text{in the sense of distributions in } \mathbb{R}^3, \\ \nabla \bar{H}_m \in L^2(\mathbb{R}^3, \mathbb{R}^3). \end{cases} \quad (1.4)$$

The first term in (1.3) is usually called the exchange energy while the second is called the anisotropy energy and the third is called the demagnetization energy. One can consider an infinite cylindrical domain $\Omega = G \times \mathbb{R}$ and configurations which do not depend on the last coordinate. The original model is then reduced to a two-dimensional one, where the energy, up to a rescaling, has the form

$$E_\varepsilon(m) := \varepsilon \int_G |\nabla m|^2 dx + \frac{1}{\delta_\varepsilon} \int_G W(m) dx + \frac{1}{\varepsilon} \int_{\mathbb{R}^2} |\nabla \bar{H}_{m'}|^2 dx, \quad (1.5)$$

where $G \subset \mathbb{R}^2$ is a bounded domain, $m = (m_1, m_2, m_3) : G \rightarrow S^2$ stands for the magnetization, $m' := (m_1, m_2) \in \mathbb{R}^2$ denotes the first two components of m , $\delta_\varepsilon > 0$ and $\bar{H}_{m'} : \mathbb{R}^2 \rightarrow \mathbb{R}$ is defined, as before, by

$$\begin{cases} \Delta \bar{H}_{m'} = \operatorname{div} \{ \chi_G m' \} & \text{in the sense of distributions in } \mathbb{R}^2, \\ \nabla \bar{H}_{m'} \in L^2(\mathbb{R}^2, \mathbb{R}^2). \end{cases} \quad (1.6)$$

Note that in the case $\delta_\varepsilon = \varepsilon$ (i.e. the anisotropy and the demagnetization energies have the same order as $\varepsilon \rightarrow 0$) the energy-functionals in (1.3) and (1.5) are special cases of the energy in (1.1).

In this work, using the technique developed in [24] and [25], we construct the upper and lower bounds, as $\varepsilon \downarrow 0$, for a general energy of the form (1.1) for functions $\phi \in BV$, under certain conditions on the set $\mathcal{M}$. In particular, our upper bound improves, in general, the one obtained in [23].

1.2. Survey of the previous results

In various applications one is led to study the Γ -limit, as $\varepsilon \rightarrow 0^+$, of functionals depending on a small parameter ε . In the first type of problems the functional I_ε , defined for functions $\psi : \Omega \rightarrow \mathbb{R}^m$, takes the form

$$I_\varepsilon(\psi) = \int_{\Omega} \left\{ \varepsilon |\nabla \psi(x)|^2 + \frac{1}{\varepsilon} W(\psi(x), x) \right\} dx, \quad (1.7)$$

$$\text{or more generally,} \quad I_\varepsilon(\psi) = \int_{\Omega} \left\{ \frac{1}{\varepsilon} G(\varepsilon^n \nabla \psi^n, \dots, \varepsilon \nabla \psi, \psi, x) + \frac{1}{\varepsilon} W(\psi, x) \right\} dx, \quad (1.8)$$

where $W(\cdot)$ and $G(\cdot)$ are nonnegative functions and $G(0, \dots, 0, \psi, x) \equiv 0$. In the second type of problems functional I_ε is defined for $u : \Omega \rightarrow \mathbb{R}^k$ and has the form

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