Absolutely Norm attaining Paranormal operators

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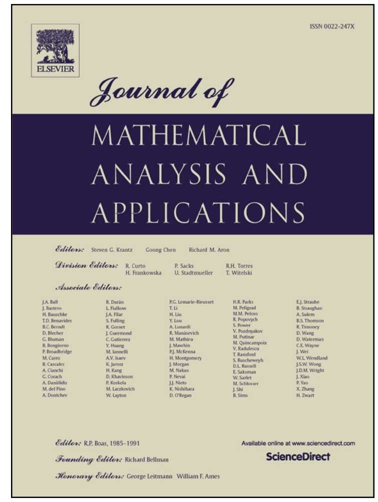

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ABSOLUTELY NORM ATTAINING PARANORMAL OPERATORS

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ABSTRACT. A bounded linear operator $T : H_1 \to H_2$, where H_1, H_2 are Hilbert spaces is said to be norm attaining if there exists a unit vector $x \in H_1$ such that $\|Tx\| = \|T\|$. If for any closed subspace M of H_1, the restriction $T|_M : M \to H_2$ of T to M is norm attaining, then T is called an absolutely norm attaining operator or $\mathcal{AN}$-operator. We prove the following characterization theorem:
A positive operator T defined on an infinite dimensional Hilbert space H is an $\mathcal{AN}$-operator if and only if the essential spectrum of T is a single point and $[m(T), m_e(T))$ contains atmost finitely many points. Here $m(T)$ and $m_e(T)$ are the minimum modulus and essential minimum modulus of T.

As a consequence we obtain a sufficient condition under which the $\mathcal{AN}$-property of an operator implies $\mathcal{AN}$-property of its adjoint.

We also study the structure of paranormal $\mathcal{AN}$-operators and give a necessary and sufficient condition under which a paranormal $\mathcal{AN}$-operator is normal.

1. Introduction

In this article we continue the study of absolutely norm attaining operators of the earlier work from [7]. The class of absolutely norm attaining operators is introduced in [3] and further the detailed study of these operators is appeared in [6, 10, 7].

In the present article first we prove a characterization theorem for positive $\mathcal{AN}$-operators. In general if T is an $\mathcal{AN}$-operator, it may not be true that T^* is also an $\mathcal{AN}$-operator (See [10, Example 6.3] for more details). We give a sufficient condition under which this result holds true.

Next, we study the structure of absolutely norm attaining paranormal operators. Specifically, we show that if T is a paranormal $\mathcal{AN}$-operator, then there exists pairs (H_α, U_α), where H_α is a reducing subspace of T and U_α is an isometry on H_α such that

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