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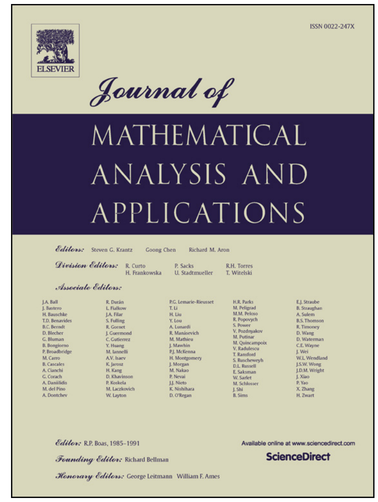


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Zhongwei Tang, Lushun Wang

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# Segregated vector solutions with multi-scale spikes for nonlinear coupled elliptic systems

ZHONGWEI TANG *and LUSHUN WANG †
School of Mathematical Sciences,
Beijing Normal University, Beijing, 100875, P.R. of China

**Abstract**

In this paper, we consider the following nonlinear coupled elliptic system

$$\begin{cases} -\varepsilon^2\Delta u + P(x)u = \mu_1 u^3 + \beta u v^2 & \text{in } \mathbb{R}^N, \\ -\varepsilon^2\Delta v + Q(x)v = \mu_2 v^3 + \beta u^2 v & \text{in } \mathbb{R}^N, \\ u > 0, \quad v > 0 & \text{in } \mathbb{R}^N, \\ u \to 0, \quad v \to 0 & \text{as } |x| \to +\infty, \end{cases} \qquad (\mathcal{A}_\varepsilon)$$

where $\varepsilon > 0$ is a parameter, $\mu_1, \mu_2 > 0$ and $\beta > 0$ are constants, $P(x)$ and $Q(x)$ are two nonnegative, smooth functions which have different non-degenerate critical points and separated zero sets. Due to the Liapunov-Schmidt reduction method and the Maximum Principle, we show that when $\beta$ less than a small positive number, there exist $\varepsilon_0 > 0$ such that for any $0 < \varepsilon < \varepsilon_0$, $(\mathcal{A}_\varepsilon)$ has a segregated vector solution $(u_\varepsilon, v_\varepsilon)$ with $u_\varepsilon$ is trapped in a neighborhood of non-degenerate critical points of $P(x)$ and also the zero sets of $P(x)$, $v_\varepsilon$ is trapped in a neighborhood of non-degenerate critical points of $Q(x)$ and also the zero sets of $Q(x)$. Moreover the amplitudes of the $u_\varepsilon$ ( res $v_\varepsilon$) around the non-degenerate critical points and the zero sets of $P(x)$( res $Q(x)$) are of a different order of $\varepsilon$. As far as the authors know, these multi-scale solutions for system have not been obtained before.

**Keywords**:Nonlinear coupled elliptic systems; Liapunov-Schmidt reduction methods; Segregated vector solutions, Multi-scale spikes.
**AMS Subject Classification**: 35J60, 35J20

## 1 Introduction and main results

In this paper, we are concerned with the solitary wave solutions of time-dependent coupled nonlinear Schrödinger equations given by

$$\begin{cases} -i\frac{\partial\Phi_1}{\partial t} = \varepsilon^2\Delta\Phi_1 - V_1(x)\Phi_1 + \mu_1|\Phi_1|^2\Phi_1 + \beta\Phi_1|\Phi_2|^2 & \text{for } y \in \mathbb{R}^N, t > 0, \\ -i\frac{\partial\Phi_2}{\partial t} = \varepsilon^2\Delta\Phi_2 - V_2(x)\Phi_2 + \mu_2|\Phi_2|^2\Phi_2 + \beta|\Phi_1|^2\Phi_2 & \text{for } y \in \mathbb{R}^N, t > 0, \\ \Phi_1(y,t) \in \mathbb{C}, \quad \Phi_2(y,t) \in \mathbb{C}, & \\ \Phi_1(y,t) \to 0, \quad \Phi_2(y,t) \to 0 & \text{as } |y| \to +\infty, t > 0 \end{cases} \tag{1.1}$$

where $N = 1, 2, 3$, $\varepsilon$, $\mu_1$, $\mu_2$ are positive constants, $\beta$ is a coupling constant and $V_1(x), V_2(x)$ are two smooth potentials.

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†Corresponding author:tangzw@bnu.edu.cn