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# Harnack inequality and pinching estimates for anisotropic curvature flow of hypersurfaces

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ABSTRACT

We obtain a differential Harnack inequality for anisotropic curvature flow of convex hypersurfaces in Euclidean space with its speed given by a curvature function of homogeneity degree one in a certain class, and restrictions depending only on the initial data and the anisotropic factor which reflects the influence of the ambient space. Moreover, the pinching estimate for such flows is derived from the maximum principle for tensors.

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## 1. Introduction

As a tool to study parabolic equations on manifolds, especially curvature flows, the differential Harnack inequality developed by Hamilton and many others [14,19–21] etc., commonly relies on the maximum principle for tensors. Their works prompted by the earlier pioneering work by Li and Yau [25] have been crucial to understanding the curvature flows and their solitons. To mention some of the literature, one can find these for the mean curvature flow in [21,22], for the Ricci flow in [7,12,10,18,19,22,26], and for the curvature flows of hypersurfaces in [2,8,9,11,13,16,24,27].

In this paper following from the earlier work [23], we consider the motion of convex hypersurfaces by a curvature function of homogeneity degree one with some concavity conditions multiplied by an anisotropic factor depending only on the ambient manifold $\mathbb{R}^{n+1}$. One may view this anisotropic factor as the influence of the ambient manifold. Usually the anisotropy in literature refers to the dependence on the normal vector to the moving hypersurfaces (see [3,15,29] for example), not on the ambient space like in our case. In the

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course of proving the main theorem on the Harnack inequality, a pinching estimate for anisotropic curvature flows is obtained when the curvature function lies within a certain class of smooth functions depending on the principal curvature. Such a class of curvature flows has been studied in detail by Andrews and others (see [1,2,4–6,8,9,16]).

Our work is motivated by the works of Andrews [2,4]. More recently, in [8], this has been generalised to the differential Harnack inequality on sphere for the flow moving by $p$-power of a strictly monotone, homogeneous of degree one and convex curvature function, where it was proved using the standard parametrisation. In [2], the computation was carried out in Gauss map parametrisation, and as the anisotropic factor in this work depends on the ambient space, we take the former of the two parametrisations.

Let $\psi : \mathbb{R}^{n+1} \to \mathbb{R}$ be a smooth function with $\inf_{\mathbb{R}^{n+1}} \psi > 0$ and we call it an anisotropic factor. We consider the family of immersions $\mathbf{X} : M^n \to \mathbb{R}^{n+1}$, where $M^n$ is a closed manifold, and the motion of the hypersurfaces is governed by the equation

$$\frac{\partial \mathbf{X}}{\partial t} = -\psi(\mathbf{X}) F(\mathcal{W}) \boldsymbol{\nu} = -\widetilde{F}(\mathcal{W}, \mathbf{X}) \boldsymbol{\nu}, \tag{1.1}$$

where $F$ is a function depending on the Weingarten map $\mathcal{W}$ and $\boldsymbol{\nu}$ is the outward unit normal to $\Sigma_t := \mathbf{X}(M)$.

For the speed of the flow in (1.1), we consider the curvature function $F$ within a certain class of smooth symmetric function on $\Gamma_+ = \{(\lambda_1, \cdots, \lambda_n) \in \mathbb{R}^n \,|\, \lambda_i > 0 \text{ for } i = 1, \cdots, n\}$. Throughout the paper, the indices in alphabet run from 1 to $n$. For a symmetric matrix $A$, one may write $F(A) = f(\boldsymbol{\lambda}(A))$, where $\boldsymbol{\lambda}(A) = (\lambda_1, \cdots, \lambda_n)$ is the map which takes $A$ to its eigenvalues $\lambda_j$. Thus for $A = \mathcal{W} = (h^i_j)$, one has

$$F(\mathcal{W}) = f(\lambda_1, \cdots, \lambda_n).$$

**Definition 1.1.** For a smooth function $f$ defined on the cone $S_+$ of positive definite symmetric matrices, we say $f$ is inverse-concave if

$$F_*(A) = f_*(\lambda_1, \cdots, \lambda_n) := f(\lambda_1^{-1}, \cdots, \lambda_n^{-1})^{-1},$$

is concave for any $A \in S_+$.

As in [4,6], we shall consider concave and inverse-concave $F$, and introduce the class of functions to which the speed $F$ of the flow (1.1) belong.

**Definition 1.2.** Denote by $\dot{f}^k$ the derivative of $f(\boldsymbol{\lambda}(A))$ with respect to $\lambda_k$, where $A \in S_+$. The function $F$ is said to be in $\mathcal{V}_n$ if

(i) $f > 0$ and $\dot{f}^k > 0$ for $i = 1, \cdots, n$,
(ii) $f(\boldsymbol{\lambda})$ and $f_*(\boldsymbol{\lambda})$ are concave in $\boldsymbol{\lambda}$, and
(iii) $f$ is homogeneous of degree one.

The examples of functions in $\mathcal{V}_n$, as discussed in [4], include $H$, $R^{1/2}$ and the $k$-th root of the elementary symmetric function $\sigma_k$ of principal curvatures for $1 \leq k \leq n$, where $H$ and $R$ denote the mean curvature and the scalar curvature, respectively. For $F$ in $\mathcal{V}_n$, one may see that the flow in (1.1) has a maximal time $T > 0$. For the case of $\psi \equiv 1$, the $C^\infty$-convergence of the rescaled flow to a round sphere was proved in [1]. Given a strict convex initial hypersurface, we shall consider a strictly convex family of immersions throughout the paper.

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