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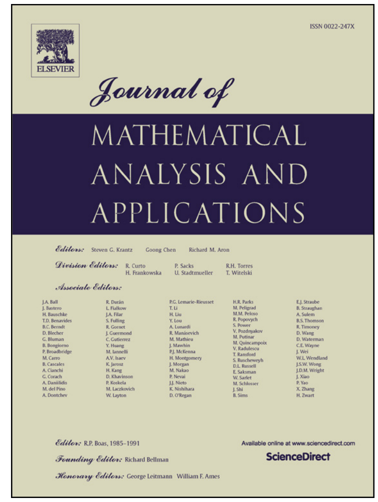

PII: S0022-247X(18)30365-2
DOI: https://doi.org/10.1016/j.jmaa.2018.04.058
Reference: YJMAA 22212

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 31 March 2018

GLOBAL ASYMPTOTICAL BEHAVIOR AND SOME NEW BLOW-UP CONDITIONS OF SOLUTIONS TO A THIN-FILM EQUATION

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Abstract. We consider a thin-film equation. By exploiting the boundary condition and the variational structure of the equation, we consider the global existence and blow-up of the solutions. For the global solutions, we study the asymptotic behavior, and for the blow-up solutions, we study the upper and lower bounds of the blow-up time. Especially, we have derived the necessary and sufficient conditions for the solution blowing up in finite time when the initial energy $J(u_0) \le d$, where d is the mountain-pass level. For $J(u_0) > d$, we also study the conditions for the solution exists globally and blows up in finite time, and we also obtain some necessary and sufficient conditions for the solution blowing up in finite time when the initial energy at arbitrary level. Furthermore, we study the decay behavior of both the global solutions and its corresponding energy functional, and the decay ratios are given specially. The results generalize the former studies on this equation, such as the papers [Nonlinear Anal., 147:96–109, 2016], [Z. Angew. Math. Phys., 68(4):Art. 89, 17, 2017] and [J. Math. Anal. Appl., 458(1):9–20, 2018].

1. Introduction

In this paper, we consider the following fourth-order evolution with p-laplace term:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \Delta^2 u - \Delta_p u = |u|^{q-2}u - \dfrac{1}{|\Omega|}\displaystyle\int_\Omega |u|^{q-2}u dx, & x \in \Omega, t > 0, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial \Delta u}{\partial \nu} = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ $(n \ge 1)$ with smooth boundary $\partial\Omega$, ν is the unit external normal direction on $\partial\Omega$,

$$\begin{cases} \Delta^2 u := \Delta(\Delta u) = \displaystyle\sum_{i,j=1}^{n} u_{x_i x_i x_j x_j}, \\ \Delta_p u := \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) = \displaystyle\sum_{j=1}^{n}\left[\left(\sum_{i=1}^{n}|u_{x_i}|^2\right)^{\frac{p-2}{2}} u_{x_j}\right]_{x_j}, \end{cases}$$

The research has been supported by Basic and Advanced Research Project of CQC-STC grant cstc2016jcyjA0018 and NSFC grant 11201380.
Keywords: Thin-film equation; Blow-up; Exponentially decay.
MSC: 35B40; 35K35; 35K58.