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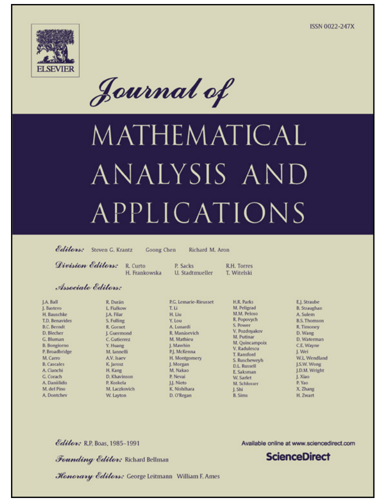

Global attractors of some predator-prey reaction-diffusion systems with density-dependent diffusion and time-delays

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Global Attractors of Some Predator-Prey Reaction-Diffusion Systems with Density-Dependent Diffusion and Time-Delays

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Abstract

This paper deals with a two-species and a three-species predator-prey reaction diffusion systems where the diffusion coefficients are density dependent and time-delays are involved in the reaction functions. The diffusion terms are of porous medium type which are degenerate and the boundary conditions are of Neumann type. The aim of the paper is to investigate the asymptotic behavior of the time-dependent solution in relation to various steady-state solutions of the system. This includes the existence of a unique positive classical solution, global attraction of a steady-state solution, and stability or instability of various semitrivial solutions and the positive steady-state solution.

Keywords: predator-prey models, quasi-linear reaction diffusion, time-delay, global attractor, upper and lower solutions

MSC: primary 35K57, 35K65; secondary 35B40, 92D25

1 Introduction

Predator-prey reaction diffusion systems have been extensively investigated in the literature and one of the most important concerns is the dynamics of the system, including the stability or instability of various steady-state solutions. However, most of the discussions in the earlier literature are for semilinear reaction diffusion equations where the diffusion coefficients are density independent and there is no time-delay in the reaction functions (cf. [5, 6, 9, 12, 20, 21, 27]). The works in [1–4, 8, 13, 25, 26] investigate a two- or a three-species predator-prey model with time-delays while those in [7, 14, 23, 24] treat some systems with density-dependent diffusion. In this paper we study two predator-prey reaction diffusion systems where the diffusion coefficients are density dependent and time delays are also involved in the reaction functions. Our first model is the one-predator one-prey quasi-linear system given in the form

$$\begin{array}{ll}
\partial u/\partial t - D_1(x)\,\Delta u^{m_1} = u\,(a_1 - b_1 u - c_1 v_{\tau_2}) & \\
\partial v/\partial t - D_2(x)\,\Delta v^{m_2} = v\,(-a_2 + b_2 u_{\tau_1} - c_2 v) & (t>0,\quad x\in\Omega) \\
\partial u/\partial\nu = \partial v/\partial\nu = 0 & (t>0,\quad x\in\partial\Omega) \\
u\,(t,x) = \eta_1\,(t,x)\,, & (\tau_1 \le t \le 0,\quad x\in\Omega) \\
v\,(t,x) = \eta_2\,(t,x) & (\tau_2 \le t \le 0,\quad x\in\Omega)
\end{array} \tag{1.1}$$

where Δ is the Laplacian, Ω is a bounded domain in $\mathbb{R}^p$ with boundary $\partial\Omega$ $(p = 1, 2, \ldots)$, $\partial/\partial\nu$ denotes the outward normal derivative on $\partial\Omega$, $u_{\tau_1} = u\,(t-\tau_1, x)$, $v_{\tau_2} = v\,(t-\tau_2, x)$ for some constants $\tau_1 \ge 0$, $\tau_2 \ge 0$, and for each $i = 1, 2$, m_i, a_i, b_i, and c_i are positive-constants with $m_i \ge 1$ and D_i and η_i are positive functions satisfying the conditions in Hypothesis (H_1) in Section 2. Our second model is given by the three-species model

$$\begin{array}{ll}
\partial u/\partial t - D_1(x)\,\Delta u^{m_1} = u\,(a_1 - b_1 u - c_1 v_{\tau_2}) & \\
\partial v/\partial t - D_2(x)\,\Delta v^{m_2} = v\,(-a_2 + b_2 u_{\tau_1} - c_2 v - d_2 w_{\tau_3}) & (t>0,\quad x\in\Omega) \\
\partial w/\partial t - D_3(x)\,\Delta w^{m_3} = w\,(-a_3 + b_3 v_{\tau_2} - c_3 w) & \\
\partial u/\partial\nu = \partial v/\partial\nu = \partial w/\partial\nu = 0 & (t>0,\quad x\in\partial\Omega) \\
u\,(t,x) = \eta_1\,(t,x)\,, & (-\tau_1 \le t \le 0,\quad x\in\Omega) \\
v\,(t,x) = \eta_2\,(t,x)\,, & (-\tau_2 \le t \le 0,\quad x\in\Omega) \\
w\,(t,x) = \eta_3\,(t,x) & (-\tau_3 \le t \le 0,\quad x\in\Omega)
\end{array} \tag{1.2}$$