J. Math. Anal. Appl. 464 (2018) 1383–1407

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

A new error analysis of nonconforming Bergan's energy-orthogonal element for the Extended Fisher–Kolmogorov equation ☆

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A R T I C L E I N F O

Article history:
Received 8 May 2017
Available online 28 April 2018
Submitted by R.G. Durán

Keywords:
EFK equation
Nonlinear
Nonconforming FEM
Optimal error estimate

A B S T R A C T

A nonconforming finite element method (FEM for short) is proposed and analyzed for the fourth order Extended Fisher–Kolmogorov (EFK for short) equation by employing the Bergan's energy-orthogonal plate element. Because the shape function and its first derivatives of this element are discontinuous at the element's vertices, which is quite different from the conventional finite elements used in the existing literature, a series of novel approaches including some a priori bounds and interpolation function splitting are developed to present a new error analysis for deriving optimal estimates of order $O(h)$ in energy norm for both the semi-discrete and backward Euler fully-discrete schemes. At last, numerical experiments are also provided to verify the theoretical analysis.

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1. Introduction

Considering the following fourth order parabolic EFK equation:

$$\begin{cases} u_t + \gamma \triangle^2 u - \triangle u + f(u) = 0, & (X,t) \in \Omega \times (0,T], \\ u(X,0) = u^0(X), & X \in \Omega, \\ u = \dfrac{\partial u}{\partial \mathbf{n}} = 0, & (X,t) \in \partial\Omega \times (0,T], \end{cases} \tag{1}$$

where $X = (x,y)$, $\Omega \subset R^2$ is a bounded convex polygon domain with boundary $\partial\Omega$, $\mathbf{n}$ is the unit outward normal vector, $T > 0$, $f(u) = u^3 - u$ is a nonlinear term and γ is a positive constant. When $\gamma = 0$ in (1), we obtain the standard FK equation which occurs in the study of front propagation into unstable states (*cf.* [1]).

☆ This work was supported by the National Natural Science Foundation of China (Nos. 11701523, 11671369, 11626221), and the Foundation of Henan Educational Committee (No. 17A110012).

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https://doi.org/10.1016/j.jmaa.2018.04.059
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The problem (1) is first proposed in [12] as a higher order model equation for spatial patterns in bistable systems. It also has a lot of applications in the theory of propagation of domain walls in nematic liquid crystals, traveling waves in reaction–diffusion systems and so on (*cf.* [1,17,38]). There have been many works concentrated on this equation (*cf.* [8–11,13,15,19,21,23,34,36]). From the numerical point of view, some finite difference schemes of the EFK equation can be found in [19,21], and a C^1 conforming FEM was established in [10] and optimal error estimates depended on $\frac{1}{\gamma}$ were obtained for both the semi-discrete and fully-discrete schemes. Further, by using some C^0 conforming elements, a fully discrete C^0 interior penalty FEM, mixed FEM, H^1-Galerkin mixed FEM and a linearized mixed FEM have been presented in [11,13,15,36] respectively.

It is well known that the degree of piecewise polynomials of a C^1 conforming finite element space must be very high to meet C^1 smoothness requirement. For example, it needs at least a incomplete 5-degree polynomial with 18 parameters for a triangular element (Bell element), or a bi-cubic polynomial with 16 parameters for a rectangular element (Bogner–Fox–Schmit (BFS) element) (*cf.* [7]). Apparently the dimension of the related finite element space is fairly large and its structure is rather complicated, which may cause some computational difficulties. To reduce the order of polynomials and degrees of freedom on each element, nonconforming elements are an attractive option for the smoothness requirement on finite element functions is weakened. It should be pointed out that the weaken continuity of nonconforming elements will make convergence analysis and error estimates more difficult. The readers are referred to [24,33,35] and [7,29] for convergence condition and a summary account of nonconforming FEMs respectively. Up to now, nonconforming FEMs have been widely applied to the fourth order problems, such as elliptic problem, singular perturbation problem, displacement obstacle problem of clamped plate and so on (*cf.* [2,4–6,16,18,22,26–28,30,37]).

However, as far as we know, there is no report on nonconforming FEMs for problem (1). An important reason is that the convergence analysis in the existing literature depend heavily on some a priori bounds of finite element solutions. Specifically, by using Lyapunov functional and Sobolev imbedding theorem directly, some a priori bounds in L^∞-norm for C^1 conforming finite element solutions and L^q-norm $(1 \le q < \infty)$ for C^0 conforming case were established in [10] and [11,13,15,36] respectively. Obviously, these analyses are no longer valid to nonconforming elements without C^0 continuity.

In this paper, as a first attempt, a new error analysis of the Bergan's energy-orthogonal element for approximating the EFK equations (1) is discussed. This element is constructed based on an energy-orthogonal free formulation and its degrees of freedom are simple (*cf.* [3,14,31,32]). But the shape function and its first derivatives are not continuous at the element's vertices for its very special interpolation procedure. Because of such high discontinuity of this element, some novel approaches have to be developed so as to get optimal error estimates for the semi-discrete and backward Euler fully-discrete schemes. Especially some a priori bounds in L^{2k}-norm $(k = 1, 2, 3 \cdots)$ are obtained by introducing an auxiliary problem, and the consistency error term is estimated with an idea of splitting the finite element function into two parts. At the same time, numerical experiments are carried out to verify the theoretical analysis.

The remainder of this paper is organized as follows. In the next section, the Bergan's energy-orthogonal element is described briefly. In Section 3, some a priori bounds of approximation solutions are presented with help of an auxiliary problem. Then by taking interpolation operator instead of the complicated projection used in [10], optimal error estimates in the energy norm for the semi-discrete scheme are successfully established. In Section 4, the backward Euler fully-discrete scheme is considered. In the last section, numerical results are provided to confirm the theoretical analysis.

The following standard notations for the Sobolev spaces will be used: $H^m(\Omega)$ with norm $\|\cdot\|_m$ and semi-norm $|\cdot|_m$, $H^m(K)$ with norm $\|\cdot\|_{m,K}$ and semi-norm $|\cdot|_{m,K}$, $L^2(\Omega)$ with norm $\|\cdot\|_0$, $L^p(\Omega)$ with norm $\|\cdot\|_{0,p}$ and $L^\infty(\Omega)$ with norm $\|\cdot\|_{0,\infty}$ respectively, where $m > 0$ is an integer, $1 \le p < \infty$ is a real number. Besides, let $P_k(K)$ be the space consisting of piecewise polynomials of degree k on element K.

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