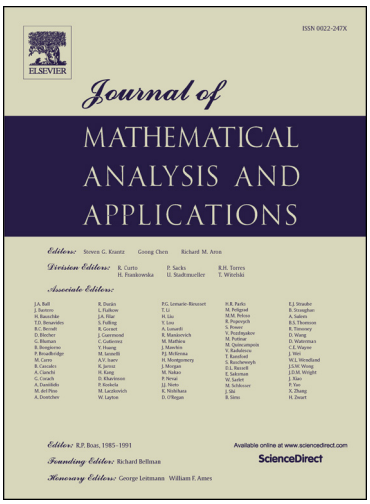
Accepted Manuscript



Matrix-valued Jensen's inequality and its application to eigenvalue analysis

Yongdo Lim, Miklós Pálfia

PII: S0022-247X(18)30379-2
DOI: https://doi.org/10.1016/j.jmaa.2018.04.072
Reference: YJMAA 22226

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 16 January 2018

Please cite this article in press as: Y. Lim, M. Pálfia, Matrix-valued Jensen's inequality and its application to eigenvalue analysis, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2018.04.072

MATRIX-VALUED JENSEN'S INEQUALITY AND ITS APPLICATION TO EIGENVALUE ANALYSIS

YONGDO LIM AND MIKLÓS PÁLFIA

Abstract. We first establish Jensen's inequality for lower semicontinuous convex functions from a Hadamard space into an "ordered" Hadamard space. We apply Jensen's inequality to matrix valued integrable functions on probability measure spaces and provide new results on matrix analysis and eigenvalue analysis by discovering a variety of Lipschitz convex functions related to eigenvalue maps and Löwner ordering of positive definite matrices.

2010 Mathematics Subject Classification. Primary 47A64, 47A56 Secondary 15A15, 15A24

Key words and phrases. Positive definite matrix, Jensen's inequality, Wasserstein distance, contractive barycenter, majorization, eigenvalue inequalities.

1. Introduction and main results

Let (M, d) be a Hadamard space and let $x\#y$ denote the unique midpoint between x and y in M. The midpoint map can be extended to a quite general setting of probability measures on M, so called the canonical barycentric map β_M via the least squares problem;

$$\beta_M(\mu) = \arg\min_{z\in M} \int_M [d^2(z,a) - d^2(x,a)]d\mu(a)$$

for $\mu \in \mathcal{P}^1(M)$, the space of integrable Borel probability measures on M. One of remarkable results of the canonical barycentric map is Jensen's inequality. K.-T. Sturm [23] established Jensen's inequality for "real valued" lower semicontinuous convex functions on Hadamard spaces (alternatively CAT(0) or NPC spaces). Two entirely different proofs presented by Sturm are also readily applicable for maps taking values in ordered Hadamard spaces. Here a Hadamard space is "ordered" if there

Date: April 30, 2018.