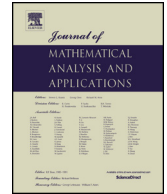




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# A blow-up result for a quasilinear chemotaxis system with logistic source in higher dimensions

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## ABSTRACT

In this paper we consider the quasilinear chemotaxis system

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (u\nabla v) + f(u), & x \in \Omega, t > 0, \\ 0 = \Delta v - \mu(t) + u, & x \in \Omega, t > 0, \end{cases}$$

with homogeneous Neumann boundary conditions in a bounded domain  $\Omega \subset \mathbb{R}^n$  with  $n \geq 2$ , where  $\chi > 0$ ,  $\mu(t) := \frac{1}{|\Omega|} \int_{\Omega} u(x, t) dx$  and  $f \in C([0, \infty)) \cap C^1((0, \infty))$  is a logistic source of the form  $f(s) = as - bs^\kappa$  with  $a \geq 0, b > 0, \kappa > 1$  and  $s \geq 0$ , and the diffusion  $D \in C^2([0, \infty))$  is supposed to satisfy

$$D(s) \geq D_0 s^{-m} \text{ for all } s > 0$$

with some  $D_0 > 0$  and  $m \in \mathbb{R}$ . Given any  $b > 0$ , when the logistic source is strong enough in the sense that

$$\kappa > m + 3 - \frac{4}{n+2} \text{ and } \kappa > 2,$$

it is shown that for any initial data  $u_0 \in C^0(\bar{\Omega})$  and  $n \geq 2$  the problem possesses a unique global bounded classical solution. However, when

$$D(s) = D_0 s^{-m} \text{ for all } s > 0$$

with  $\frac{4}{n} - 1 < m \leq 0$  in the sense that  $n \geq 5$ , and the effect of logistic source is weaker in the sense that

$$\kappa \in \left( 1, \frac{(3-m)n-2}{2n-2} \right),$$

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it is shown that for arbitrary prescribed  $M_0 > 0$  there exists initial data  $u_0 \in C^\infty(\bar{\Omega})$  satisfying  $\int_{\Omega} u_0 = M_0$  such that the corresponding solution  $(u, v)$  of the system blows up in finite time in a ball  $\Omega = B_0(R) \subset \mathbb{R}^n$  with some  $R > 0$ . This result extends the blow-up arguments of the Keller–Segel chemotaxis model with logistic cell kinetics in Winkler [39] to more general quasilinear case. Moreover, since there is a gap in the proof of Zheng et al. [46], it also presents modified results for the mistake.

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## 1. Introduction

We consider the following quasilinear chemotaxis system with logistic source

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (u\nabla v) + f(u), & x \in \Omega, t > 0, \\ 0 = \Delta v - \mu(t) + u, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x), & x \in \Omega, \\ \int_{\Omega} v(x, t) dx = 0, & t > 0, \end{cases} \tag{1.1}$$

in a bounded domain  $\Omega \subset \mathbb{R}^n (n \geq 2)$  with a given nonnegative smooth function  $u_0$ , where  $\chi > 0$  is a fixed parameter. This model involves two variables: the density of cells, denoted by  $u$ , the density of chemoattractant, represented by  $v$ , the latter being secreted only by cells themselves. The function  $\mu(t)$  denotes the time-dependent spatial mean of  $u(\cdot, t)$  in the sense that

$$\mu(t) := \frac{1}{|\Omega|} \int_{\Omega} u(x, t) dx \text{ for all } t > 0. \tag{1.2}$$

The logistic source  $f \in C^0([0, \infty)) \cap C^1((0, \infty))$  fulfills  $f(0) \geq 0$  and the diffusion function  $D \in C^2([0, \infty))$  satisfies

$$D(s) > 0, \quad s \geq 0 \text{ and } D(s) \geq D_0 s^{-m} \text{ for all } s > 0 \tag{1.3}$$

with some  $D_0 > 0$  and  $m \in \mathbb{R}$ .

The theoretical study of such above processes by means of cross-diffusive parabolic systems of the considered form was initiated by Keller and Segel in their seminal work (see [14]), which can be obtained by letting  $D \equiv 1, f \equiv 0$ , and by replacing the second equation in (1.1) with the parabolic case  $v_t = \Delta v - v + u$ . It is proved that the solutions never blow up if  $n = 1$  [25] or  $n = 2$  and  $\int_{\Omega} u_0 < \frac{4\pi}{\chi}$  [6,24], whereas  $n = 2$  and  $\int_{\Omega} u_0 > \frac{4\pi}{\chi}$  [10,27] or in higher dimensions  $n \geq 3$  [37,40] the solutions may blow up in finite or infinite time. Since the chemicals diffuse much faster than cells, a parabolic–elliptic system was derived for more general nonlinear diffusive function

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (u\nabla v), & x \in \Omega, \quad t > 0, \\ 0 = \Delta v - v + u, & x \in \Omega, \quad t > 0 \end{cases} \tag{1.4}$$

with  $D$  given by (1.3), where the second equation in (1.4) can be viewed as the following form

$$0 = \Delta v - \mu(t) + u, \quad x \in \Omega, \quad t > 0, \tag{1.5}$$

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