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A blow-up result for a quasilinear chemotaxis system with logistic source in higher dimensions

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ABSTRACT

In this paper we consider the quasilinear chemotaxis system

$$\begin{cases} u_t = \nabla\cdot(D(u)\nabla u) - \chi\nabla\cdot(u\nabla v) + f(u), & x\in\Omega, t>0,\\ 0 = \Delta v - \mu(t) + u, & x\in\Omega, t>0, \end{cases}$$

with homogeneous Neumann boundary conditions in a bounded domain $\Omega\subset\mathbb{R}^n$ with $n\geq 2$, where $\chi>0$, $\mu(t) := \frac{1}{|\Omega|}\int_\Omega u(x,t)dx$ and $f\in C([0,\infty))\cap C^1((0,\infty))$ is a logistic source of the form $f(s) = as - bs^\kappa$ with $a\geq 0, b>0$, $\kappa>1$ and $s\geq 0$, and the diffusion $D\in C^2([0,\infty))$ is supposed to satisfy

$$D(s)\geq D_0 s^{-m} \quad \text{for all } \ s>0$$

with some $D_0>0$ and $m\in\mathbb{R}$. Given any $b>0$, when the logistic source is strong enough in the sense that

$$\kappa > m+3-\frac{4}{n+2} \quad \text{and} \quad \kappa>2,$$

it is shown that for any initial data $u_0\in C^0(\bar{\Omega})$ and $n\geq 2$ the problem possesses a unique global bounded classical solution. However, when

$$D(s) = D_0 s^{-m} \quad \text{for all } \ s>0$$

with $\frac{4}{n}-1<m\leq 0$ in the sense that $n\geq 5$, and the effect of logistic source is weaker in the sense that

$$\kappa\in\left(1, \frac{(3-m)n-2}{2n-2}\right),$$

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it is shown that for arbitrary prescribed $M_0 > 0$ there exists initial data $u_0 \in C^\infty(\bar{\Omega})$ satisfying $\int_\Omega u_0 = M_0$ such that the corresponding solution (u, v) of the system blows up in finite time in a ball $\Omega = B_0(R) \subset \mathbb{R}^n$ with some $R > 0$. This result extends the blow-up arguments of the Keller–Segel chemotaxis model with logistic cell kinetics in Winkler [39] to more general quasilinear case. Moreover, since there is a gap in the proof of Zheng et al. [46], it also presents modified results for the mistake.

1. Introduction

We consider the following quasilinear chemotaxis system with logistic source

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (u \nabla v) + f(u), & x \in \Omega, t > 0, \\ 0 = \Delta v - \mu(t) + u, & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \\ \int_\Omega v(x,t)dx = 0, & t > 0, \end{cases} \tag{1.1}$$

in a bounded domain $\Omega \subset \mathbb{R}^n (n \geq 2)$ with a given nonnegative smooth function u_0, where $\chi > 0$ is a fixed parameter. This model involves two variables: the density of cells, denoted by u, the density of chemoattractant, represented by v, the latter being secreted only by cells themselves. The function $\mu(t)$ denotes the time-dependent spatial mean of $u(\cdot, t)$ in the sense that

$$\mu(t) := \frac{1}{|\Omega|} \int_\Omega u(x,t)dx \text{ for all } t > 0. \tag{1.2}$$

The logistic source $f \in C^0([0,\infty)) \cap C^1((0,\infty))$ fulfills $f(0) \geq 0$ and the diffusion function $D \in C^2([0,\infty))$ satisfies

$$D(s) > 0,\ s \geq 0 \text{ and } D(s) \geq D_0 s^{-m} \text{ for all } s > 0 \tag{1.3}$$

with some $D_0 > 0$ and $m \in \mathbb{R}$.

The theoretical study of such above processes by means of cross-diffusive parabolic systems of the considered form was initiated by Keller and Segel in their seminal work (see [14]), which can be obtained by letting $D \equiv 1$, $f \equiv 0$, and by replacing the second equation in (1.1) with the parabolic case $v_t = \Delta v - v + u$. It is proved that the solutions never blow up if $n = 1$ [25] or $n = 2$ and $\int_\Omega u_0 < \frac{4\pi}{\chi}$ [6,24], whereas $n = 2$ and $\int_\Omega u_0 > \frac{4\pi}{\chi}$ [10,27] or in higher dimensions $n \geq 3$ [37,40] the solutions may blow up in finite or infinite time. Since the chemicals diffuse much faster than cells, a parabolic–elliptic system was derived for more general nonlinear diffusive function

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (u \nabla v), & x \in \Omega, \quad t > 0, \\ 0 = \Delta v - v + u, & x \in \Omega, \quad t > 0 \end{cases} \tag{1.4}$$

with D given by (1.3), where the second equation in (1.4) can be viewed as the following form

$$0 = \Delta v - \mu(t) + u, \quad x \in \Omega, \quad t > 0, \tag{1.5}$$