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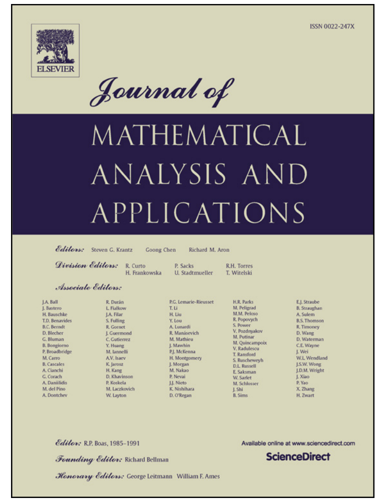

Generalized Bloch spaces, Integral means of hyperbolic harmonic mappings in the unit ball

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GENERALIZED BLOCH SPACES, INTEGRAL MEANS OF HYPERBOLIC HARMONIC MAPPINGS IN THE UNIT BALL

JIAOLONG CHEN

Abstract. In this paper, we investigate the properties of hyperbolic harmonic mappings in the unit ball $\mathbb{B}^n$ in $\mathbb{R}^n$ ($n \geq 2$). Firstly, we establish necessary and sufficient conditions for a hyperbolic harmonic mapping to be in the Bloch space $\mathcal{B}(\mathbb{B}^n)$ and the generalized Bloch space $\mathcal{L}_{\infty,\omega}\mathcal{B}^0_{\alpha,\mathrm{a}}(\mathbb{B}^n)$, respectively. Secondly, we discuss the relationship between the integral means of hyperbolic harmonic mappings and that of their gradients. The obtained results are the generalizations of Hardy and Littlewood's related ones in the setting of hyperbolic harmonic mappings. Finally, we characterize the weak uniform boundedness property of hyperbolic harmonic mappings in terms of the quasihyperbolic metric.

1. Introduction and main results

For $n \geq 2$, let $\mathbb{B}^n(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| < r\}$, $\mathbb{S}^{n-1}(x_0, r) = \partial\mathbb{B}^n(x_0, r)$ and $\overline{\mathbb{B}}^n(x_0, r) = \mathbb{B}^n(x_0, r) \cup \mathbb{S}^{n-1}(x_0, r)$. In particular, we write $\mathbb{B}^n = \mathbb{B}^n(0, 1)$, $\mathbb{S}^{n-1} = \mathbb{S}^{n-1}(0, 1)$ and $\overline{\mathbb{B}}^n = \mathbb{B}^n \cup \mathbb{S}^{n-1}$.

The purpose of this paper is to consider the hyperbolic harmonic mappings whose definition is as follows.

Definition 1.1. A mapping $u = (u_1, \cdots, u_n) \in C^2(\mathbb{B}^n, \mathbb{R}^n)$ is said to be *hyperbolic harmonic* if

$$\Delta_h u = (\Delta_h u_1, \cdots, \Delta_h u_n) = 0,$$

that is, for each $j \in \{1, \cdots, n\}$, u_j satisfies the hyperbolic Laplace equation

$$\Delta_h u_j = 0,$$

where

$$\Delta_h u_j(x) = (1 - |x|^2)^2 \Delta u_j(x) + 2(n-2)(1 - |x|^2) \sum_{i=1}^{n} x_i \frac{\partial u_j}{\partial x_i}(x). \tag{1.1}$$

We refer to [4, 15, 19, 32, 40, 41, 42] for basic properties of this class of mappings. For convenience, in the following of this paper, we always use the notation $\Delta_h u = 0$ to mean that $u = (u_1, \cdots, u_n)$ is hyperbolic harmonic in $\mathbb{B}^n$.

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