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Limit theorems for non-Markovian marked dynamic contagion processes ☆

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ABSTRACT

In this paper, we study a general dynamic contagion process that includes several standard point processes as special cases. We have developed (i) the corresponding large deviation principle; (ii) the corresponding law of large numbers; (iii) the corresponding central limit theorem, all of which are critical in describing the essential of this type of processes, and are not available in current literature. The proposed model provides a broader framework in the study of contagion processes in various applications. In particular, we obtain the finite-horizon and infinite-horizon ruin probability asymptotics for the risk model with marked dynamic contagion claim arrivals as an application.

1. Introduction

The systematic risk and contagion described by point processes play a crucial role in the study of financial crisis. Many recent study starts to pay attention to this research topic since the Asian financial crisis of the late 90s as well as the recent financial crisis of 2007–2008. From the mathematical perspective, a point process with its intensity dependence on the process itself can provide a quite effective model to capture the contagion phenomenon of those so-called clustering 'bad' events, for example, see [14], and [6] whose studies are based on the self-excited Hawkes process. Moreover, the default intensity can be influenced externally by multiple common factors, such as various sector or market-wide events (see, e.g., [5] as well as [16]). Most recently, a dynamic contagion process with exponential decays has been studied in literature (e.g., see [1]) in which it introduces a new point process with both the externally excited and self-excited dependence structures. More precisely, the approach is characterized by a point process $\bar{N}_t \equiv \{T_k\}_{k\geq 1}$ on $\mathbb{R}^+$ with the intensity in a form of

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$$\lambda_t = a + (\lambda_0 - a)e^{-\delta t} + \sum_{i\geq 1} Y_i^{(1)} e^{-\delta(t-T_i^{(1)})} 1_{\{T_i^{(1)}\leq t\}} + \sum_{j\geq 1} Y_j^{(2)} e^{-\delta(t-T_j)} 1_{\{T_j \leq t\}}, \tag{1}$$

where $T^{(1)}$ is a standard Poisson process with parameter $\rho > 0$, independent of other processes as well as both $Y_i^{(1)}$ and $Y_j^{(2)}$, which are i.i.d. random marks following suitable distribution functions $F_1(y)$ and $F_2(y)$, respectively, and the corresponding generator is given by

$$\begin{aligned} \mathcal{A}f(\lambda, n, t) = \tfrac{\partial f}{\partial t} - \delta(\lambda - a)\tfrac{\partial f}{\partial \lambda} + \rho \left(\int_0^\infty f(\lambda + y, n, t)dF_1(y) - f(\lambda, n, t)\right) \\ +\lambda \left(\int_0^\infty f(\lambda + y, n + 1, t)dF_2(y) - f(\lambda, n, t)\right). \end{aligned} \tag{2}$$

In general, the dynamic contagion process can be considered as a special case (without the diffusion terms) of the general affine point processes appeared in [4], with the infinitesimal generator specified by (2). This model contains the characteristics of the two well-known point processes in the literature: (1) Cox process with Poisson shot-noise intensity; (2) Hawkes process with exponentially decaying intensity.

Yan and Gao [21] obtain the sample path large and moderate deviation principles by introducing an exponential martingale for a Cox risk process with Poisson shot noise intensity, in which they also provide an estimate of ruin probability. Hefter and Herzwurm [13] study the strong (pathwise) approximation of Cox–Ingersoll–Ross processes and prove the positive convergence rate for the full parameter range including the accessible boundary regime. For a risk process with the arrival of claims modeled by a dynamic contagion process, under the net profit condition $c > \frac{\mu_H \rho + a\delta}{\delta - \mu_G}\mu_Z$, the ruin probabilities have been studied in [2]. There have been some progress made in the direction of asymptotic results other than the large time limits, for instance, when the exciting function (external-exciting function g and self-exciting function h) are exponential, the intensity process and the pair (N_t, λ_t) are Markovian. Gao and Zhu [10,8] study the large deviations and functional limit theorems of Markovian affine point processes with large initial intensity. Gao and Zhu [9] obtain the asymptotic results of nonlinear Hawkes process. The limit theorems have also been studied for an extension of linear Hawkes processes and Cox–Ingersoll–Ross processes in [25], which has applications in short interest rate models in finance. Zhu [26] considered Markovian case for nonlinear Hawkes processes when exciting function is exponential or in more general, a sum of exponential ones, and characterize the large deviations. Yao and Xiao [22] consider a generalized stochastic model associated with affine point processes and study the large deviations for large initial intensity and the corresponding functional law of large numbers and functional central limit theorems. For the non-Markovian case, Karabash and Zhu [15] obtain large deviations for linear marked Hawkes processes. Gao and Zhu [7] study precise deviations for non-Markovian Hawkes processes for large time consideration.

In this paper we consider a more general dynamic contagion process. More specifically, the intensity at time t is set to be

$$\lambda_t = \nu(t) + \sum_{\tau_i^{(1)} < t} g(t - \tau_i^{(1)}) + \sum_{\tau_j < t} h(t - \tau_j), \tag{3}$$

where $\tau^{(1)}$ is Poisson process with parameter $\rho > 0$ and τ is the dynamic contagion process. And $g, h : \mathbb{R}^+ \to \mathbb{R}^+$ are real functions. Thus

$$N_t \equiv \{\tau_k\}_{k\geq 1} \tag{4}$$

is the corresponding dynamic contagion process and

$$N_t^{(1)} \equiv \{\tau_k^{(1)}\}_{k\geq 1} \tag{5}$$

is the corresponding Poisson process. We also consider dynamic contagion process with random marks.

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