Doctopic: Optimization and Control ARTICLE IN PRESS YJMAA:22187

J. Math. Anal. Appl. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Global Carleman estimates for the linear stochastic Kuramoto–Sivashinsky equations and their applications ☆

Peng Gao

School of Mathematics and Statistics, and Center for Mathematics and Interdisciplinary Sciences, Northeast Normal University, Changchun 130024, PR China

ARTICLE INFO

Article history:
Received 3 September 2017
Available online xxxx
Submitted by X. Zhang

Keywords:
Stochastic Kuramoto–Sivashinsky equation
Carleman estimate
Observability
Null controllability
Unique continuation property

ABSTRACT

In this paper, we establish two new global Carleman estimates for the linear stochastic Kuramoto–Sivashinsky equations. The first one is for the backward linear stochastic Kuramoto–Sivashinsky equation. Based on this estimate and the duality argument, we obtain the null controllability of the forward linear stochastic Kuramoto–Sivashinsky equation by three controls, one is an internal control in the diffusion term and the others are boundary controls. The second one is for the forward linear stochastic Kuramoto–Sivashinsky equation. According to this estimate, we obtain a unique continuation property for the forward linear stochastic Kuramoto–Sivashinsky equation.

1. Introduction

The stochastic Kuramoto–Sivashinsky (KS) equation appears in the study of dynamic roughening in sputter-eroded surfaces and, in principle, in any physical system modeled by the deterministic KS equation in which the relevance of time-dependent noise as, e.g., fluctuations in a flux or thermal fluctuations, can be argued for. In [7] the early and late time dynamics of the erosion model were numerically studied with the conclusion that they are the same as those obtained from the stochastic KS equation [15]. In [17], the authors confirmed this result by showing analytically that the stochastic KS equation yields the continuum description of the erosion model.

In this paper, we establish new global Carleman estimates for the backward linear stochastic KS equation

$$\begin{cases} dz - (kz_{xx} + z_{xxxx})dt = (pz + qZ + h)dt + Zdw & \text{in } Q, \\ z(0,t) = 0 = z(1,t) & \text{in } (0,T), \\ z_x(0,t) = 0 = z_x(1,t) & \text{in } (0,T), \\ z(x,T) = z_T(x) & \text{in } I \end{cases} \tag{1.1}$$

☆ This work is supported by NSFC Grant (11601073), NSFC Grant (11701078), China Postdoctoral Science Foundation (2017M611292), the Fundamental Research Funds for the Central Universities (2412017QD002).

E-mail address: gaopengjilindaxue@126.com.

https://doi.org/10.1016/j.jmaa.2018.04.033
0022-247X/© 2018 Elsevier Inc. All rights reserved.

and the forward linear stochastic KS equation

$$\begin{cases} dz + (kz_{xx} + z_{xxxx})dt = (pz + f)dt + (qz + g)dw & \text{in } Q, \\ z(0,t) = 0 = z(1,t) & \text{in } (0,T), \\ z_x(0,t) = 0 = z_x(1,t) & \text{in } (0,T), \\ z(x,0) = z_0(x) & \text{in } I \end{cases} \tag{1.2}$$

with suitable coefficients p and q. In (1.1) and (1.2), $I = (0,1), T > 0, Q = I \times (0,T), k > 0$ is the antidiffusion parameter.

In recent years, many efforts have been devoted to studying the Carleman estimates for stochastic partial differential equations:

- stochastic transport equations [24],
- stochastic heat equation [2,29,21,20],
- stochastic wave equation [31],
- stochastic KdV equation [9],
- stochastic Kuramoto–Sivashinsky equations [14],
- stochastic Schrödinger equation [23],
- stochastic fourth order Schrödinger equations [13]
- stochastic Kawahara equation [11]
- ······

Through this paper, we make the following assumptions:

(H1) We denote by $L^2(I)$ the space of all Lebesgue square integrable functions on I. The inner product on $L^2(I)$ is

$$(u,v)_{L^2(I)} = \int_I uv dx,$$

for any $u, v \in L^2(I)$.

$H^s(I)(s \geq 0)$ are the classical Sobolev spaces of functions on I. The definition of $H^s(I)$ can be found in [18].

(H2) Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete filtered probability space on which a one-dimensional standard Brownian motion $\{w(t)\}_{t\geq 0}$ is defined such that $\{\mathcal{F}_t\}_{t\geq 0}$ is the natural filtration generated by $w(\cdot)$, augmented by all the P-null sets in $\mathcal{F}$. Let H be a Banach space, and let $C([0,T];H)$ be the Banach space of all H-valued strongly continuous functions defined on $[0,T]$. We denote by $L^2_{\mathcal{F}}(0,T;H)$ the Banach space consisting of all H-valued $\{\mathcal{F}_t\}_{t\geq 0}$-adapted processes $X(\cdot)$ such that $E(\|X(\cdot)\|^2_{L^2(0,T;H)}) < \infty$, with the canonical norm; by $L^\infty_{\mathcal{F}}(0,T;H)$ the Banach space consisting of all H-valued $\{\mathcal{F}_t\}_{t\geq 0}$-adapted bounded processes; and by $L^2_{\mathcal{F}}(\Omega; C([0,T];H))$ the Banach space consisting of all H-valued $\{\mathcal{F}_t\}_{t\geq 0}$-adapted continuous processes $X(\cdot)$ such that $E(\|X(\cdot)\|^2_{C([0,T];H)}) < \infty$, with the canonical norm.

(H3) Unless otherwise stated, C stands for a generic positive constant whose value can change from line to line. If it is essential, the dependence of a constant C on some parameters, say "·", will be written by $C(\cdot)$.

(H4) Let $\psi(x) = (x - x_0)^2 + \delta_0$, where δ_0 is a positive constant such that $\psi \geq \frac{3}{4}\|\psi\|_{L^\infty(I)}$ and $x_0 > 1$. For any given positive constants λ and μ, we set $\rho(x,t) = \frac{e^{\mu\psi(x)} - e^{\frac{3}{2}\|\psi\|_{L^\infty(I)}\mu}}{t(T-t)}$, $l = \lambda\rho$, $\theta = e^l$ and $\varphi(x,t) = \frac{e^{\mu\psi(x)}}{t(T-t)}, \forall (x,t) \in Q$.

(H5) $p, q, a, b \in L^\infty_{\mathcal{F}}(0,T;L^\infty(I))$.