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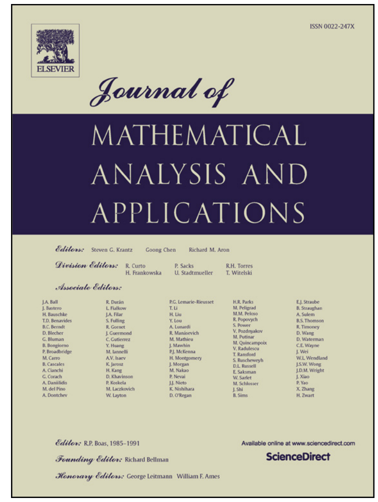

Global regularity to the Cauchy problem of the 3D heat conducting incompressible Navier–Stokes equations

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Global regularity to the Cauchy problem of the 3D heat conducting incompressible Navier–Stokes equations

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Abstract. This paper concerns the global regularity to the Cauchy problem of the 3D heat conducting incompressible Navier–Stokes equations with density–temperature–dependent viscosity and vacuum. Through the t–weighted a priori estimates, we establish the global existence and decay of strong solutions provided the initial energy is suitably small. It should be noted that the absolute temperature can be large initially.
Keywords: global strong solution; decay; density–temperature–dependent viscosity; vacuum.

MSC: 35B40; 35B45; 76D03; 76D05

1 Introduction

We consider the following heat conducting incompressible fluid in $\mathbb{R}^3$:

$$\begin{cases} \rho_t + \mathrm{div}(\rho u) = 0, \\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) - \mathrm{div}(2\mu(\rho,\theta) d) + \nabla P = 0, \\ c_v[(\rho\theta)_t + \mathrm{div}(\rho u \theta)] - \mathrm{div}(\kappa \nabla \theta) = 2\mu(\rho,\theta)|d|^2, \\ \mathrm{div} u = 0, \end{cases} \tag{1.1}$$

together with the initial and boundary conditions

$$\begin{cases} (\rho, u, \theta)(x, \cdot) \to 0, \quad \text{as } |x| \to \infty, \\ (\rho, u, \theta)(x, 0) = (\rho_0, u_0, \theta_0)(x). \end{cases} \tag{1.2}$$

Here we denote by ρ, u, θ and P the unknown density, velocity, absolute temperature and pressure for the fluid, respectively; $d = \frac{1}{2}[\nabla u + (\nabla u)^T]$ expresses the deformation tensor; c_v and κ are positive constants; $\mu(\rho,\theta)$ is the viscosity coefficient and assumed to satisfy

$$0 < \underline{\mu} \le \mu(\rho,\theta) \in C^1(\mathbb{R}^2). \tag{1.3}$$

There is much literature about the mathematical theory of the incompressible Navier–Stokes equations without heat conduction. In the case that μ is a positive constant and initial density has a positive lower bound, [3, 4, 15, 17] established global weak solutions and local strong solutions to the incompressible Navier–Stokes equations. As for the local and global

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