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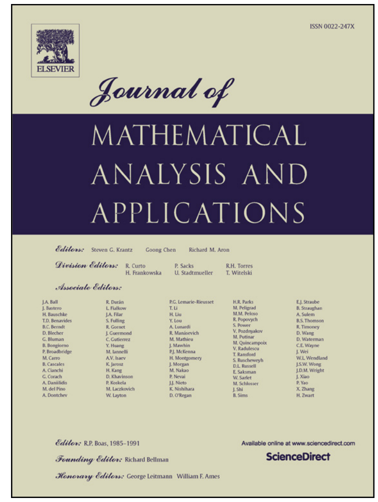

General criteria for curves to be simple

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GENERAL CRITERIA FOR CURVES TO BE SIMPLE

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Abstract. We extend previous results for parametrized curves in euclidean space to be simple. The new condition depends as before on Ahlfors' Schwarzian and considers a *conformal metric* on a given interval and the new diameter. We derive some applications, among which we find Becker type conditions that depend on a pre-Schwrazian.

1. Introduction

The purpose of this paper is to extend results in [5], where the use of Sturm comparison and Ahlfors' Schwarzian for curves led to sufficient conditions for parametrized curves in euclidean space to be simple. In many cases, the condition was sharp. By considering a "conformal metric" on an interval, we derive here a more general condition of the same type that takes into account the modified diameter of the interval. The theorem fills in the gaps when the former condition was not optimal. In addition, suitable choices of the conformal factor give rise to criteria that depend on a pre-Schwarzian derivative, and analogues of criteria for holomorphic mappings in the disk due to Ahlfors, Becker, and Epstein [2], [3], [9].

We begin with a brief account on Ahlfors' Schwarzian for curves. In [1] the author generalizes the Schwarzian to cover $f:(a,b)\to\mathbb{R}^n$ by separately defining analogues of the real and imaginary parts $\mathrm{Re}\{Sf\},\mathrm{Im}\{Sf\}$ of the Schwarzian of a locally injective mapping f. For parametrized curves with $f'\neq 0$ he defined

$$S_1 f = \frac{\langle f', f'''\rangle}{|f'|^2} - 3\frac{\langle f', f''\rangle^2}{|f'|^4} + \frac{3}{2}\frac{|f''|^2}{|f'|^2}\,, \tag{1.1}$$

and

$$S_2 f = \frac{f'\wedge f'''}{|f'|^2} - 3\frac{\langle f', f''\rangle}{|f'|^4} f'\wedge f''\,, \tag{1.2}$$

respectively. Here, $\langle\, ,\,\rangle$ denotes the standard inner product, and for $\vec{a},\vec{b}\in\mathbb{R}^n$, $\vec{a}\wedge\vec{b}$ is the antisymmetric bivector with components $(\vec{a}\wedge\vec{b})_{ij}=a_ib_j-a_jb_i$ and norm $\left[\sum(a_ib_j-a_jb_i)^2\right]^{1/2}$. Ahlfors indicated that he was led to these seemingly esoteric

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