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Decay rates of the compressible viscoelastic flows with electric potential

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ABSTRACT

We study the global existence and decay rates of strong solutions to the three dimensional compressible viscoelastic flows with an electric potential. The energy method for high frequency parts combined with a spectral analysis for low frequency parts is used to obtain the global solution and its decay rate (in time). We show that the density of the solution converges to its equilibrium state at the L^2-decay rate $(1+t)^{-\frac{3}{4}-\frac{1}{2}}$ when the initial perturbation admits some small assumptions. Compared to the compressible viscoelastic flows Hu and Wu (2013) [13], our results imply that the rotating effect of the electric field affects the dispersion of fluids and enhances the time decay rate of the density.

1. Introduction

In this paper, we consider three-dimensional compressible viscoelastic flows with an electric potential

$$\begin{cases} \rho_t + \nabla\cdot(\rho u) = 0, \\ (\rho u)_t + \nabla\cdot(\rho u\otimes u) + \nabla P(\rho) - \mu\Delta u - (\lambda+\mu)\nabla\nabla\cdot u = \alpha\nabla\cdot(\rho F F^T) + \rho\nabla\Phi, \\ F_t + u\cdot\nabla F = \nabla u F, \\ \Delta\Phi = \rho - \varrho, \end{cases} \tag{1.1}$$

for $(x,t)\in\mathbb{R}^3\times[0,+\infty)$. Here, $\rho>0, u\in\mathbb{R}^3$ and $F\in M^{3\times 3}$ denote the density, the velocity and the deformation gradient, respectively. The constants μ and λ are the shear viscosity and the bulk viscosity coefficients of the fluid, respectively, which satisfy

$$\mu>0,\ 2\mu+3\lambda\geq 0.$$

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The self-consistent electric potential $\Phi = \Phi(x,t)$ is coupled with the density through the Poisson equation. The constant $\varrho > 0$ denotes the background doping profile. The corresponding elastic energy is chosen to be the special form of the Hookean linear elasticity, elastic energy (Hookean linear elasticity)

$$W(F) = \frac{\alpha}{2}|F|^2 + \frac{1}{\rho}\int_0^\rho P(s)\mathrm{d}s,$$

where $\alpha > 0$ is the speed of propagation of shear waves. The pressure $P = P(\rho)$ is a smooth function with $P'(\rho) > 0$ for $\rho > 0$.

We pose the initial data of this system (1.1) as follows:

$$(\rho, u, F)|_{t=0} = (\rho_0(x), u_0(x), F_0(x)), \quad \text{for } x \in \mathbb{R}^3, \tag{1.2}$$

which satisfies

$$\operatorname{div}(\rho_0 F_0^T) = 0, \qquad F^{lk}(0)\nabla_l F^{ij}(0) = F^{lj}(0)\nabla_l F^{ik}(0). \tag{1.3}$$

The condition (1.3) can be preserved by the flow. This is a standard result which can be found in [11,27] and no proof will be given here. In this paper, we focus on the small perturbed solution to (1.1) around the trivial equilibrium state solution $(\varrho, \mathfrak{u}, \mathfrak{F})$. Without loss of generality, we choose the constant state $(\varrho, \mathfrak{u}, \mathfrak{F}) = (1, 0, I)$ where I is the identity matrix in $M^{3\times 3}$. For simplicity, we assume that $P'(1) = 1$.

It is necessary to review some results about the system (1.1) and related models. The system (1.1) without the electric potential is a compressible viscoelastic model which has been introduced in [3,6,18,28]. When there is no external (or internal) force involved, there are many results about the global existence of smooth (or strong) solution to the compressible viscoelastic flows. For multi-dimensional case, the local existence theory of a strong solution was established by Hu and Wang in [10]. The global existence of a solution was obtained in [11,27] for the initial value problem and in [12,26] for the initial boundary value problem. For the large time behavior of the global strong solution, Hu and Wu in [13] get the optimal decay rates

$$\|\nabla^k(\rho - 1, u, F - I)\|_{L^2(\mathbb{R}^3)} \le C(1+t)^{-\frac{3}{4}-\frac{k}{2}}, \quad \text{for } 0 \le k \le 1.$$

Recently, the authors in [34] achieved the optimal decay rates in Negative Sobolev space. For the incompressible viscoelastic flows, we refer to [1,2,15,16,21,8,14,17,25,37,20,22,23] and references therein.

When the electric potential is taken into account, the system (1.1) is a coupling system between Navier–Stokes–Poisson (NSP) equations and the deformation gradient. To our best knowledge, there are very few mathematical results on the global existence of smooth (or strong) solution to the system (1.1), especially for the analysis of the large time behavior. We show in this paper that the density of the solution converges to its equilibrium states at the L^2-decay rates $(1+t)^{-\frac{3}{4}-\frac{k+1}{2}}$, which is faster than the L^2-decay rates $(1+t)^{-\frac{3}{4}-\frac{k}{2}}$ for the compressible viscoelastic flows in [13]. The dispersion effect of the electric field contributes to enhance the decay rate of the density. It should be noted that for the Navier–Stokes–Poisson equations, a quite similar phenomena is shown, see [32]. The decay rate of the solutions to the NSP system has been investigated extensively, see for instance [7,19,33,38,30,35,31,36]. By the way, it is worth pointing out that the viscosity term in system (1.1) has significant effects on the global existence and large time behavior of solutions. When the viscosity term is absence, system (1.1) becomes a repulsive Hookean elastodynamics, the related mathematical analysis become extremely hard. Recently, the global existence of classical solutions to the repulsive Hookean elastodynamics was obtained in [9] by Hu and Masmoudi.

Our main purpose of this paper is to establish the global existence in time and large time behavior of solutions to the Cauchy problem (1.1)–(1.2). By employing the low frequency and high frequency decomposition, we divide the solution $\mathbb{U} = (\tilde{\rho}, \tilde{u}, \tilde{F})(x,t)$ into two parts, that is, the high frequency part $\mathbb{U}_h$ and