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Dynamics of a nonlocal multi-type SIS epidemic model with seasonality

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ABSTRACT

In this paper, we propose and study a multi-type SIS nonlocal epidemic model with seasonality. We first establish the existence of the spreading speed c_* and the non-existence of periodic traveling waves with wave speed $c < c_*$. Then, we prove the existence and asymptotic behavior of the periodic traveling fronts with speed $c > c_*$. Finally, the existence of the critical periodic traveling wave fronts is established by using a limiting argument. To overcome the difficulty of lack of compactness of solution maps of the nonlocal system with respect to compact open topology, we show that the solution sequence is pre-compact in $L_{\text{loc}}(\mathbb{R}^2, \mathbb{R}^m)$.

1. Introduction

To model the spatial spread of a deterministic epidemic in multi-types of population, Rass and Radcliffe [12] proposed and studied a multi-type SIS epidemic model. Consider m populations, each consisting of susceptible and infectious individuals. Let the numbers of susceptible and infectious individuals in the ith population at location x and time t be $S_i(x,t)$ and $I_i(x,t)$, respectively. Suppose that the infection rate of a type i susceptible by a type k infectious individual is $\mu_{i,k} \geq 0$ and the corresponding contact distribution is $p_{i,k}(\cdot)$. Assume that the infectious individuals in population i return to the susceptible state at rate ν_i. Then the model is described as follows:

$$\begin{cases} \frac{\partial S_i(x,t)}{\partial t} = -S_i(x,t) \sum_{k=1}^m \mu_{i,k} \int_{\mathbb{R}} I_k(x-y,t) p_{i,k}(y) dy + \nu_i I_i(x,t), \\ \frac{\partial I_i(x,t)}{\partial t} = S_i(x,t) \sum_{k=1}^m \mu_{i,k} \int_{\mathbb{R}} I_k(x-y,t) p_{i,k}(y) dy - \nu_i I_i(x,t), \\ i = 1, \cdots, m, \ x \in \mathbb{R}, \ t > 0. \end{cases} \tag{1.1}$$

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Assume further that the population size of the ith population is σ_i, i.e. $\sigma_i = S_i(x,t) + I_i(x,t)$. Then, system (1.1) can be re-written as

$$\frac{\partial I_i(x,t)}{\partial t} = \big[\sigma_i - I_i(x,t)\big] \sum_{k=1}^{m} \mu_{i,k} \int_{\mathbb{R}} I_k(x-y,t) p_{i,k}(y)dy - \nu_i I_i(x,t),$$
$$i = 1, \cdots, m,\ x \in \mathbb{R},\ t > 0. \tag{1.2}$$

Denote $y_i(x,t) = I_i(x,t)/\sigma_i$. Then system (1.2) reduces to

$$\frac{\partial y_i(x,t)}{\partial t} = \big[1 - y_i(x,t)\big] \sum_{k=1}^{m} \sigma_k \mu_{i,k} \int_{\mathbb{R}} y_k(x-z,t) p_{i,k}(z)dz - \nu_i y_i(x,t),$$
$$i = 1, \cdots, m,\ x \in \mathbb{R},\ t > 0. \tag{1.3}$$

Rass and Radcliffe [12, Chapter 8] made a complete analysis on the global dynamics of the spatially homogeneous m-dimensional system of (1.3). The speed of first spread of infection was also obtained by using the saddle point method. Weng and Zhao [19] further established the existence of the asymptotic speed of propagation of infection and showed that it coincides with the critical wave speed for traveling wave fronts. Their results also gave an affirmative answer to an open problem presented by Rass and Radcliffe [12]. Zhang and Zhao [23] considered the spreading speed and traveling wave fronts of a spatially discrete version of (1.3). Recently, Wu and Chen [20] studied the uniqueness and stability of the traveling wave fronts of (1.3).

Note that the seasonal variation is not considered in the epidemic models (1.1) and (1.2). As pointed out by Altizer et al. [2], seasonal variations in temperature, rainfall, and resource availability are ubiquitous and can exert strong pressures on population dynamics. Therefore, it is more realistic to consider the time-periodic versions of (1.1) and (1.2). In this paper, we consider the following time-periodic and nonlocal epidemic model:

$$\frac{\partial u_i(x,t)}{\partial t} = \big[\sigma_i(t) - u_i(x,t)\big] \sum_{k=1}^{m} \mu_{i,k}(t) \int_{\mathbb{R}} u_k(x-y,t) p_{i,k}(y)dy - \nu_i(t) u_i(x,t),$$
$$i = 1, \cdots, m,\ x \in \mathbb{R},\ t > 0, \tag{1.4}$$

where $\sigma_i(t)$, $\mu_{i,k}(t)$, $\nu_i(t)$ are all T-periodic and nonnegative functions, $T > 0$ is a constant. It is clear that (1.4) is a time-periodic version of (1.2).

The purpose of this paper is to study the dynamics of system (1.4), including the global attractivity of positive $T-$periodic solution, spreading speeds, and time-periodic traveling wave fronts. Since (1.4) is a cooperative system, we can apply the monotone semi-flow theory developed in [10,11,18] to study the spreading speeds of it, say c_*. However, due to the lack of compactness of solution maps of (1.4), the abstract theory in [10,11,18] are difficult to be applied to study the existence of the periodic traveling wave fronts of (1.4). In this paper, we shall extend the monotone iteration technique and the limiting argument (cf. [3,15,26]) to the periodic and nonlocal system.

More precisely, by constructing a pair of explicit super- and sub-solution (see Lemma 3.4), we shall prove existence of the periodic traveling fronts $\Phi(x + ct, t)$ with speed $c > c_*$. The asymptotic behavior of the periodic traveling fronts is also obtained. To establish the existence of the periodic traveling fronts with speed $c = c_*$ (critical periodic traveling fronts for short), we consider a sequence of periodic traveling wave fronts $\{\Phi^{(n)}(x+c_n t, t)\}_{n\in\mathbb{N}}$ with $\{c_n\} \subset (c_*, +\infty)$ and $\lim_{n\to\infty} c_n = c_*$. Since $\{\Phi^{(n)}(x,t)\}_{n\in\mathbb{N}}$ is not compact with respect to compact open topology, we can not obtain a convergence subsequence of it. To overcome