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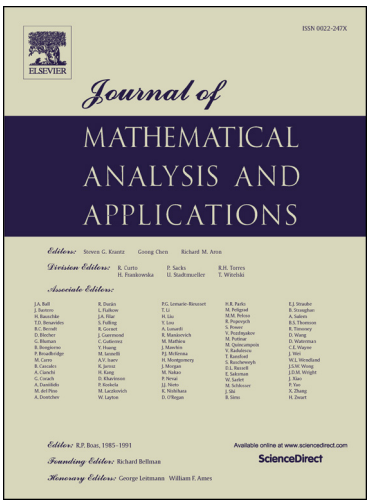

Li–Yau type and Souplet–Zhang type gradient estimates of a parabolic equation for the V-Laplacian

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Li–Yau type and Souplet–Zhang type gradient estimates of a parabolic equation for the V-Laplacian

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Abstract

In this paper, we study a parabolic equation concerning V-Laplacian on complete Riemannian manifold M:

$$(\Delta_V - q(x,t) - \partial_t)u(x,t) = A(u(x,t))$$

on $M \times [0,T]$, where V is a vector field on M and $\Delta_V \star := \Delta\star + \langle V, \nabla\star\rangle$. We give a gradient estimate of Li-Yau type for positive solutions of this equation. As corollary, we obtain a new gradient estimate in the case that $A(u) = au\log u$. We also prove a Souplet–Zhang type gradient estimate for positive bounded solutions of this equation. As corollary, we obtain a gradient estimate for positive bounded solution of

$$(\Delta_V - q(x,t) - \partial_t)u(x,t) = a(u(x,t))^\beta, \quad \beta > 1.$$

Keywords: Gradient estimate, Parabolic equation, Positive solution.
2010 MSC: 35B45, 35K55.

1. Introduction

In the seminal paper [14], P. Li and S.T. Yau studied following heat equation

$$(\Delta - q(x,t) - \partial_t)u(x,t) = 0$$

on a complete Riemannian manifold. They derived the gradient estimate for positive solutions of the above equation. Precisely, they proved

Theorem A. *Let M^n be a complete manifold with boundary ∂M. Assume $p \in M$ and let $B_{2R}(p)$ be a geodesic ball of radius $2R$ around p which does not*

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