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Fuzzification of the miscible displacement model in heterogeneous porous media

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ABSTRACT

Typical realistic models for multiphase flow in heterogeneous formations are complex and random, and must incorporate the uncertainties inherent to the mixture process. These uncertainties can be modeled using differential equations coefficients, such as hydrodynamic dispersivity. In this work, the mathematical model is expressed in terms of a nonlinear coupled system of stochastic partial differential equations; a second order elliptic equation for the pressure, and a hyperbolic-dominated transport-diffusion equation for the solvent concentration in the mixture. Besides, the longitudinal dispersion coefficient is a fuzzy number. New perspective on the quantification of uncertainty for parameter estimation problems by means of numerical simulations and membership functions is the purpose of this research. In this way, a fuzzification of the semiclassical solution numerical approximation is built. In this regard, it is proved the continuity of the function that assigns the 3-tuple comprised by longitudinal dispersion, transverse dispersion, and molecular diffusion, to the corresponding value of the semiclassical solution, at a fixed point of the domain. The continuity result along with Zadeh's Extension Principle is applied to obtain the fuzzification. The relevance of this study resides in the novelty of the methodology that considers a model parameter as a fuzzy number, meanwhile, it is usually taken as a constant in literature. Other unprecedented result lays in the discovery of the link between theoretical concepts and numerical approximations to obtain a fuzzification.

1. Introduction

To obtain a precise and detailed knowledge of the fluid flow in heterogeneous porous media phenomenon, it is necessary that the system of governing equations incorporates the uncertainties inherent to the mixture process. These uncertainties are often modeled using the coefficients of differential equations (porosity, permeability, and hydrodynamic dispersivity), which are considered as random variables in relation to the spatial position. For this reason, typical realistic models for multiphase flow in heterogeneous formations

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are expressed in terms of a nonlinear coupled system of stochastic partial differential equations, which has a strongly hyperbolic character.

Three laws are fundamental for describing miscible displacement in heterogeneous porous media. One of them is the well-known Darcy's law which is employed to govern the process known as convection, physical transport at a macroscopic level. The other is Fick's law which deals with the process of diffusion of one fluid into another. This process is due to the random motion of molecules. The law of conservation of mass is the third one.

Consider a bounded domain Ω in $\mathbb{R}^2$, the interval $J = [0, T]$, $T > 0$, and $Q_T = \Omega \times J$. A model that represents the incompressible miscible displacement of a mixture of a solvent, with concentration c, and oil in a mean free of gravitational effects, is given by the following system (see [63])

$$\mathbf{u} = -\frac{K(\mathbf{x})}{\mu(c)}\nabla p, \qquad \nabla \cdot \mathbf{u} = q^I - q^P, \tag{1}$$

$$\phi(\mathbf{x})\frac{\partial c}{\partial t} - \nabla \cdot (D(\mathbf{u})\nabla c) + \mathbf{u} \cdot \nabla c + q^I c = \widehat{q}^I. \tag{2}$$

The first equation (1) represents the mass conservation of the mixture, and the second one (2) represents the mass conservation of the solvent. In general, the concentration equation (2) is a convection-dominated parabolic equation with coefficients depending on pressure through the Darcy's velocity, $\mathbf{u} = (u_1, u_2)$. This velocity yields the volumetric convective flow rate of the mixture per unit cross-sectional area. The other elements that appear in the system are: $K = K(\mathbf{x})$, $\mathbf{x} \in \Omega$, the absolute permeability of the rock; $\mu = \mu(c)$ the viscosity of the fluid, which depends on the solvent concentration, $c = c(\mathbf{x}, t)$, $t \in J$; the pressure gradient, $\nabla p = \nabla p(\mathbf{x})$; the porosity of the medium, ϕ; q^I and q^P represent the sum of injection well source terms and the sum of production well sink terms, respectively, both sums are non-negative; $\hat{q}^I = \hat{c}q^I$, where $\hat{c}$ is the specified concentration at an injection well and the resident concentration at a producer; $D = D(\mathbf{u})$ is the diffusion–dispersion tensor, given by

$$D = D(\mathbf{u}) = \phi d_m I + \frac{d_\ell}{|\mathbf{u}|}\begin{pmatrix} u_1^2 & u_1 u_2 \\ u_1 u_2 & u_2^2 \end{pmatrix} + \frac{d_t}{|\mathbf{u}|}\begin{pmatrix} u_2^2 & -u_1 u_2 \\ -u_1 u_2 & u_1^2 \end{pmatrix}. \tag{3}$$

In tensor D, the parameters d_ℓ and d_t are respectively the longitudinal and transverse dispersion coefficients, where $d_\ell \gg d_t$ and $d_t \geq 0$, and $d_m > 0$ is the molecular diffusion coefficient. In this work, the parameter d_ℓ is considered as a fuzzy number.

The viscosity rate, also called in the literature as *mobility ratio*, is given by $M = \mu_o/\mu_s$, where μ_o is the oil viscosity and μ_s is the solvent viscosity. The viscosity of the fluid mixture is assumed to obey the quarter-power law (see [63]), then we have

$$\mu(c) = \{(1 - c) + M^{1/4}c\}^{-4}\mu_o.$$

The function μ is strictly increasing in $[0, 1]$ when $M < 1$, having μ_s as a maximum and μ_o as a minimum. When $M > 1$, the function μ is strictly decreasing in $[0, 1]$ with maximum μ_o and minimum μ_s. A sketch of the function μ is shown in Fig. 1.

As boundary conditions, it is considered the periphery of the reservoir to be impermeable, assuming "no flow" boundary conditions. If we denote by Γ the boundary of Ω, then the boundary conditions are summarized as follows:

$$\mathbf{u} \cdot \nu = 0, \;\; \mathbf{x} \in \Gamma, \tag{4}$$

$$(-D\nabla c) \cdot \nu = 0, \;\; \mathbf{x} \in \Gamma, \;\; t \in J, \tag{5}$$

where ν is the unit outward normal vector to Γ.

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