Doctopic: Partial Differential Equations ARTICLE IN PRESS YJMAA:22107

J. Math. Anal. Appl. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Pyramidal traveling fronts in a nonlocal delayed diffusion equation

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A R T I C L E I N F O

Article history:
Received 29 April 2017
Available online xxxx
Submitted by Y. Du

Keywords:
Existence
Pyramidal traveling wave solution
Reaction–diffusion equation
Nonlocal delayed

A B S T R A C T

Recently, there have been great progresses on the study of nonplanar traveling wave solutions of reaction–diffusion equations. In this paper, we study the pyramidal traveling fronts of nonlocal delayed diffusion equation in $\mathbb{R}^N$ with $N \geq 2$ by using the comparison principle and establishing super- and subsolution. Since the effect of nonlocal delay, we show that nonlocal delayed diffusion equation in N-dimensional space has a pyramidal traveling front $u(t, \mathbf{x}) = V(\mathbf{x}', x_N + st)$ toward X_N-axis for each $s > c > 0$. In particular, two-dimensional traveling curved front and three-dimensional pyramidal fronts for nonlocal delayed diffusion equation in $\mathbb{R}^2$ and $\mathbb{R}^3$ are also established, respectively. Moreover, we also extend our results to generally nonlocal delayed reaction–diffusion equation.

1. Introduction

This paper is concerned with the multi-dimensional traveling fronts of the following nonlocal delayed diffusion equation

$$\frac{\partial u}{\partial t}(\mathbf{x}, t) = D\Delta u(\mathbf{x}, t) - du(\mathbf{x}, t) + \int\limits_{\mathbb{R}} b\left(u(\mathbf{y}', y_N, t-\tau)\right) f(x_N - y_N) dy_N \tag{1.1}$$

for $\mathbf{x} = (\mathbf{x}', x^N) \in \mathbb{R}^N$, where the Laplacian Δ stands for $\sum_{i=1}^N \frac{\partial^2}{\partial x_i^2}$ and $N \geq 2$. Equation (1.1) is usually used to describe the evolution of the adult population of a single species population with two age classes and moving around in unbounded N-dimensional spatial domain, $D > 0$ and $d > 0$ denote the diffusion rate and death rate of the adult population, respectively, $\tau \geq 0$ is the maturation time for the species, $b(\cdot)$ is related to the birth function, and the function $f(\cdot) \in C^\infty(\mathbb{R}, \mathbb{R})$ satisfies

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[1] This work was partially supported by the Fundamental Research Funds for the Central Universities (310812171001), Natural Science Basic Research Plan in Shaanxi Province of China (2017JQ1014) and NSF of China (11701041).

https://doi.org/10.1016/j.jmaa.2018.03.031

Please cite this article in press as: X. Bao, J. Liu, Pyramidal traveling fronts in a nonlocal delayed diffusion equation, J. Math. Anal. Appl. (2018), https://doi.org/10.1016/j.jmaa.2018.03.031

$$f(x) \geq 0, \int_{\mathbb{R}} f(y)dy = 1 \quad \text{and} \quad \int_{\mathbb{R}} |y|f(y)dy < +\infty. \tag{1.2}$$

We refer to [10,11] for a survey of the short history of the reaction–diffusion equations with nonlocal delayed interactions. In realistic environment, the matured population at the current time t and spatial location x was born at time $t-\tau$ or earlier and might be at different spatial location. The convolution in space term is represented the nonlocal interaction in one direction and the kernel function $f(\cdot)$ is used to describe the diffusion pattern of the immature population during the maturation process, see [31] for more details and some specific form of $f(\cdot)$ obtained from integration along characteristic equations of a structured population model. Since the time delay and nonlocality play very important roles in biological and epidemiological models (see [6] and [24]), traveling wave solutions of nonlocal delayed reaction–diffusion equations/systems in one dimensional space or planar traveling wave front in high dimensional space have been widely studied, we refer to [9], [16], [18], [19], [25], [30], [31], [39], [40], [41], [46], [47] and references therein for the related results.

In this paper, we assume that the birth function $b(\cdot) \in C^1(\mathbb{R}, \mathbb{R})$ and there exists a constant $K > 0$ such that

$$b(0) = dK - b(K) = 0,$$

which implies that (1.1) has at least two spatially homogeneous equilibria 0 and K.

We state the following standing assumptions:

(A1) $b'(u) \geq 0$ for $u \in [0, K]$;
(A2) $d > \max\{b'(0), b'(K)\}$;
(A3) there exits $u^* \in (0, K)$ such that $du^* - b(u^*) = 0$, $b'(u^*) > d$ and $du - b(u) \neq 0$ for $u \in (0, u^*) \cup (u^*, K)$.

We call that equation (1.1) is of nonlocal bistable cases if $b(u)$ satisfies (A1)–(A3). A typical example of $b(u)$ is $b(u) = pu^2 e^{-\alpha u}$ with $p > 0$ and $\alpha > 0$ in a wide rang of parameters p and α, which has been widely used in the mathematical biology literature, see [31] for details. Under the assumption (A1)–(A3), Ma and Wu [19] have proved that (1.1) has an unique, monotone increasing and globally stable traveling wave front $U(x+ct)$ connecting two spatially homogeneous equilibria 0 and K with a bounded speed c, see [19, Theorem 1.1]. Let $\xi = x + ct$ for $x \in \mathbb{R}$. Here $U(\xi)$ satisfies

$$DU''(\xi) - dU(\xi) - cU'(\xi) + \int_{\mathbb{R}} b(U(\xi - c\tau - y))f(y)dy = 0 \tag{1.3}$$

and

$$U(-\infty) = 0, \quad U(+\infty) = K.$$

Following from Wang et al. [41], if (A1)–(A3) hold, there exist positive constant β_1 and C_1 such that

$$\max\{U(-\xi), |U(\xi) - 1|, |U'(\pm\xi)|, |U''(\pm\xi)|\} \leq C_1 e^{-\beta_1 \xi}, \quad \forall\, \xi \geq 0. \tag{1.4}$$

Recently, multi-dimensional traveling fronts with different shapes of reaction–diffusion equations/systems have been widely studied, for example, see [5], [12], [13], [14], [22], [23] and [37] for two-dimensional V-form front solutions; see [8], [12], [13], [38] and [45] for cylindrically symmetric traveling fronts; see [21], [32], [33], [34], [35], [36], and [15] and [44] for multi-dimensional traveling fronts with pyramidal shapes. We also refer to [1], [2], [7], [26], [28], [27], [29], [42] and [43] for multi-dimensional traveling front of time

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