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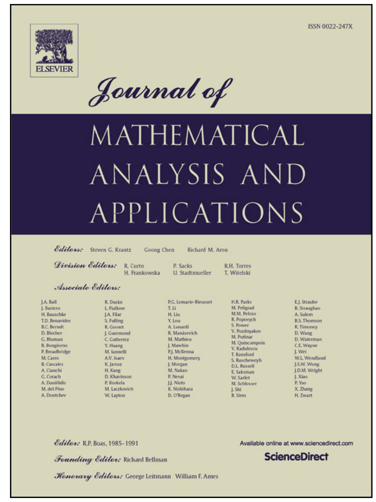

Asymptotic behaviors of solutions to a reaction–diffusion equation with isochronous nonlinearity

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ASYMPTOTIC BEHAVIORS OF SOLUTIONS TO A REACTION-DIFFUSION EQUATION WITH ISOCHRONOUS NONLINEARITY

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ABSTRACT. We study the initial boundary value problem for the reaction-diffusion equation with isochronous nonlinearity. We prove that small solutions become spatially homogeneous and is subject to the ODE part asymptotically. We also discuss blow-up of an parabolic system with quadratic nonlinearity having the origin as an uniform isochronous center.

Keywords: isochronous nonlinearity, parabolic system, invariant region, blow-up.

1. Introduction

In this paper, we prove that, if the initial value is restricted in some small neighborhood of the origin, then the solution exists globally in time, and absorbed into a ODE orbit that is periodic in time. This kind of eventually homogeneous periodic behavior has been discussed for several evolution systems. Our result can be applied to all the reaction-diffusion systems having non-degenerate isochronous center of the form:

$$u_t = d_u \Delta u - v + P(u,v), \quad v_t = d_v \Delta v + u + Q(u,v), \quad x \in \Omega,\ t > 0, \tag{1.1}$$

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, \quad x \in \partial\Omega,\ t > 0, \tag{1.2}$$

with

$$u(\cdot,0) = u_0, \qquad v(\cdot,0) = v_0, \quad x \in \Omega, \tag{1.3}$$

where d_u and d_v are positive constants, P and Q are analytic functions on $\mathbb{R}^2$ starting in at least second order terms, i.e. such that $P(0,0) = Q(0,0) = 0$ and $\partial_u P(0,0) = \partial_v P(0,0) = \partial_u Q(0,0) = \partial_v Q(0,0) = 0$, and Ω is a bounded smooth domain in $\mathbb{R}^N$ with $N \geq 1$, and ν is the outward unit normal vector. Let the vector field

$$\mathcal{X} = \{-V + P(U,V)\}\partial_U + \{U + Q(U,V)\}\partial_V$$

have a commutating analytic vector field of the form

$$\mathcal{Y} = \{U + R(U,V)\}\partial_U + \{V + S(U,V)\}\partial_V.$$

More precisely, we assume the commuting property $[\mathcal{X},\mathcal{Y}] \equiv 0$, where the bracket used here is the Lie bracket. Under this assumption, the ODE system

$$\frac{dU}{dt} = -V + P(U,V) \quad \frac{dV}{dt} = U + Q(U,V) \tag{1.4}$$

has an isochronous center at the origin, i.e. there exists a neighborhood $\mathcal{U} \subset \mathbb{R}^2$ of the origin $(U,V) = (0,0)$ such that every orbit in a punctured neighborhood $\mathcal{U} \setminus \{(0,0)\}$ is a cycle surrounding $(0,0)$, and the period of all such curves are constant 2π. It is well

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