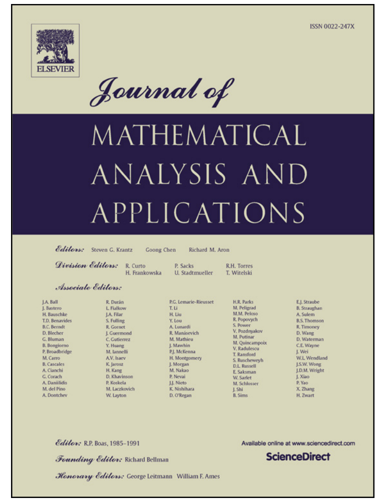

Regularization of Cauchy abstract problem for a coupled system for nonlinear elliptic equations

Nguyen Huu Can, Nguyen Huy Tuan, Vo Van Au, Le Duc Thang

PII: S0022-247X(18)30104-5
DOI: https://doi.org/10.1016/j.jmaa.2018.01.066
Reference: YJMAA 21999

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 6 April 2017

Regularization of Cauchy abstract problem for a coupled system for nonlinear elliptic equations

Nguyen Huu Can[1], Nguyen Huy Tuan[1,*], Vo Van Au[2], Le Duc Thang[3]

[1] Applied Analysis Research Group, Faculty of Mathematics and Statistics,
Ton Duc Thang University, Ho Chi Minh City, Viet Nam.
[2] Faculty of General Sciences, Can Tho University of Technology, Can Tho City, Viet Nam.
[3] Faculty of Basic Science, Ho Chi Minh City Industry and Trade College, Ho Chi Minh City, Viet Nam.

February 12, 2018

Abstract

In this paper, we consider a Cauchy problem of coupled elliptic equations. We show that the problem is ill-posed in the sense of Hadamard and a regularization is in order. Under some weak *a priori* assumptions on the exact solution, we propose a new regularization method to stabilize the problem when the source term is a global or local Lipschitz function. Furthermore, we also obtain error estimates between the regularized solution and the sought solution.

Keywords and phrases: Regularization method; Cauchy problem; Ill-posed problem; System of elliptic equations, Error estimate.

Mathematics subject Classification 2000: 35K05, 35K99, 47J06, 47H10

1 Introduction

Let $\mathcal{H}$ be a Hilbert space. Let $\mathcal{A}\colon D(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ and $\widetilde{\mathcal{A}}\colon D(\widetilde{\mathcal{A}}) \subset \mathcal{H} \to \mathcal{H}$ be linear, positive-definite, self-adjoint operators with compact inverse on $\mathcal{H}$. Let T be a positive number, we consider the problem of finding the couple (u, v), $(u, v\colon [0, T] \to \mathcal{H})$, satisfying

$$\begin{cases} u_{tt}(t) = \mathcal{A}u(t) + \mathcal{F}(u(t), v(t)), & \text{on } [0, T], \\ v_{tt}(t) = \widetilde{\mathcal{A}}v(t) + \mathcal{G}(u(t), v(t)), & \text{on } [0, T], \\ (u(0), v(0)) = (\chi_1, \chi_2), \\ (u_t(0), v_t(0)) = (\theta_1, \theta_2), \end{cases} \tag{1.1}$$

where $\chi_1, \chi_2, \theta_1, \theta_2$ are given functions in $\mathcal{H}$ and the source functions $\mathcal{F}$ and $\mathcal{G}$ will be defined later.

In practice, the data (χ_1, χ_2) and $(\theta_1, \theta_2) \in \mathcal{H} \times \mathcal{H}$ are noisy and represented by the observation data $(\chi_{1,\varepsilon}, \chi_{2,\varepsilon}), (\theta_{1,\varepsilon}, \theta_{2,\varepsilon}) \in \mathcal{H} \times \mathcal{H}$ satisfying

$$\|\chi_1 - \chi_{1,\varepsilon}\|_{\mathcal{H}} + \|\chi_2 - \chi_{2,\varepsilon}\|_{\mathcal{H}} \le \varepsilon, \quad \|\theta_1 - \theta_{1,\varepsilon}\|_{\mathcal{H}} + \|\theta_2 - \theta_{2,\varepsilon}\|_{\mathcal{H}} \le \varepsilon, \tag{1.2}$$

*Corresponding author: nguyenhuytuan@tdt.edu.vn(NguyenHuyTuan). Email of other authors: nguyenhuucan@tdt.edu.vn (Nguyen Huu Can), vvau@ctuet.edu.vn (Vo Van Au), leducthang13@gmail.com (Le Duc Thang)