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# Effect of the protection zone on coexistence of the species for a ratio-dependent predator-prey model ☆

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ABSTRACT

This paper deals with the effect of the protection zone $\Omega_0$ on coexistence of the species for a ratio-dependent predator-prey model. We obtain a critical value $\lambda_1^N(b\delta(x),\Omega)$ which is less than the well-known critical value $\lambda_1^D(\Omega_0)$ obtained in the previous literatures. Furthermore, we show that if $\lambda > \lambda_1^N(b\delta(x),\Omega)$, then the prey persists regardless of the growth rates of the predator; while if $\lambda \leq \lambda_1^N(b\delta(x),\Omega)$, then there exists a real number $\mu^*$, such that the prey is ultimately extinct when $\mu > \mu^*$. As for $\lambda < \lambda_1^N(b\delta(x),\Omega)$ and $\mu < \mu^*$, the curve of the positive steady state solutions of the model emanating from $(\lambda, 0; -c)$ ends at a singular point $(0, 0; \mu_2)$. Meantime, by using the Lyapunov–Schmidt reduction method, we obtain a fine profile of its bifurcation diagrams, and the uniqueness or multiplicity of its positive steady state solutions. In addition, as generally expected, the chances of survival of the prey will increase with the size of the protection zone.



## 1. Introduction

In the ecology and the mathematical ecology, the dynamical relationships between the predators and their preys always are one of the dominant themes due to its universal existence and importance. Although these problems can appear to be some simple mathematical models, they often are very challenging and complicated. The classical predator-prey model, due independently to Lotka and Volterra in the 1920s, only reflects the population changes due to the predation in a situation where the densities of the prey and the predator are not spatially dependent. When the spatial distributions of the two species are considered, the passive dispersal of the two species can be modelled by a diffusion operator. Thus, a natural mathe-

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matical model, which the predator and the prey are interacting and migrating in the same habitat $\Omega$, is a reaction–diffusion system as follows

$$\begin{cases} u_t - d_1\triangle u = \lambda u - u^2 - b\phi(u,v)v, & x \in \Omega, \quad t > 0; \\ v_t - d_2\triangle v = \mu v - v^2 + c\phi(u,v)v, & x \in \Omega, \quad t > 0; \end{cases} \tag{1.1}$$

where, $\Omega$ is a bounded domain in $\mathbf{R}^N$ with the smooth boundary $\partial\Omega$, $N \geq 1$; $u$ and $v$ are the densities of the prey and the predator respectively, and both the predator and the prey have the logistic growth rates; The terms $b\phi(u,v)$ and $c\phi(u,v)$ respectively account for the functional response of the predator and the conversion rates of the prey captured by the predator. In the classical Lotka–Volterra predator-prey model, it is assumed that $\phi(u,v) = u$. If the handling time of each prey is considered, then the Holling type II response $b\phi(u,v) = \frac{bu}{1+mu}$ usually is chosen [22]. However, the classical predator-prey models with $\phi(u,v) = u$ or $\frac{u}{1+mu}$ exhibit the "paradox of enrichment" and the so-called "biological control paradox" [2,3,21]. Thus, a ratio dependent functional response $b\phi(u,v) = \frac{bu/v}{1+mu/v} = \frac{bu}{mu+v}$ is introduced in [23,24,37], and the corresponding model is called as a ratio-dependent predator-prey model as follows

$$\begin{cases} u_t - d_1\triangle u = \lambda u - u^2 - \frac{buv}{mu+v}, & x \in \Omega, \quad t > 0; \\ v_t - d_2\triangle v = \mu v - v^2 + \frac{cuv}{mu+v}, & x \in \Omega, \quad t > 0. \end{cases} \tag{1.2}$$

The researches [23,24,37] showed that the ratio-dependent predator-prey model does not produce those paradox phenomena in [2,3,21]. Therefore, the ratio-dependent predator-prey model should be a more reasonable model in the prey-dependent predator-prey models.

To our knowledge, the ratio-dependent predator-prey model possesses many peculiar characters. For example, $\frac{uv}{mu+v}$ is singular and non-differential at $(u,v) = (0,0)$, and the curve of its positive solutions can be emanated from the singular point $(0,0)$ [49], which are different from those of some classical bio-mathematical models such as the Lotka–Volterra predator-prey models [26,27,46], the Holling type II predator-prey model [19] and the competition models [15,29,31,32,35]. In addition, we may re-define $\frac{buv}{u+mv} := 0$ at $(0,0)$.

For some endangered species, it is necessary that the nature reserves are established to protect these endangered species and their habitats. If a protection zone $\Omega_0$ to the prey is introduced, then the corresponding model becomes as follows

$$\begin{cases} u_t - d_1\triangle u = \lambda u - u^2 - b\delta(x)\phi(u,v)v, & x \in \Omega, \quad t > 0; \\ v_t - d_2\triangle v = \mu v - v^2 + c\phi(u,v)v, & x \in \Omega_1, \quad t > 0, \end{cases} \tag{1.3}$$

where $\Omega_0$ is a subdomain of $\Omega$ with the smooth boundary $\partial\Omega_0$. The larger region $\Omega$ is the habitat of the prey, while the predator only lives in $\Omega_1 := \Omega \setminus \overline{\Omega}_0$. Thus, $\Omega_0$ is called as the protection zone of the prey. $\delta(x)$ satisfies that $\delta(x) \equiv 0$ in $\overline{\Omega}_0$ and $\delta(x) = 1$ in $\Omega_1$, which implies that the prey enjoys the predation-free growth in $\Omega_0$. To our knowledge, Du and his coauthors [15,17,18] investigated some biomathematics models with the diffusion and the protect zone $\Omega_0$, and obtained a critical path $\lambda_1^D(\Omega_0)$ of the persistence/extinction of the prey. Oeda [38] and Wang and Li [43,44] studied effects of the cross-diffusion on some predator-prey models with the protect zones. Cui and his coauthors [10] studied strong Allee effect of a predator-prey model with a protection zone. The pioneer Professor Lopez-Gomez and his coauthors investigated some biomathematics models with the crowing terms and the protection zones, and obtained many prominent results [20,30,31].

In (1.2), if $\lambda < b$, then the prey is ultimately extinct for any sufficient large $\mu$. To the persistence of the prey, the nature reserves must be introduced into (1.2). Thus, in this paper, we will investigate the following predator-prey model with a protection zone